

**THE OPTIMAL RISK PREMIUM OF BTL(BUILD-TRANSFER-LEASE)
PROJECT: THEORETICAL AND EMPIRICAL INVESTIGATION**

By

Soojin Park

THESIS

Submitted to

KDI School of Public Policy and Management

in partial fulfillment of the requirements

for the degree of

DOCTOR OF PHILOSOPHY

IN DEVELOPMENT POLICY

2015

**THE OPTIMAL RISK PREMIUM OF BTL(BUILD-TRANSFER-LEASE)
PROJECT: THEORETICAL AND EMPIRICAL INVESTIGATION**

By

Soojin Park

THESIS

Submitted to
KDI School of Public Policy and Management
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY
IN DEVELOPMENT POLICY

2015

Professor Man CHO

**THE OPTIMAL RISK PREMIUM OF BTL(BUILD-TRANSFER-LEASE)
PROJECT: THEORETICAL AND EMPIRICAL INVESTIGATION**

By

Soojin Park

THESIS

Submitted to

KDI School of Public Policy and Management

in partial fulfillment of the requirements

for the degree of

DOCTOR OF PHILOSOPHY
IN DEVELOPMENT POLICY

Committee in charge:

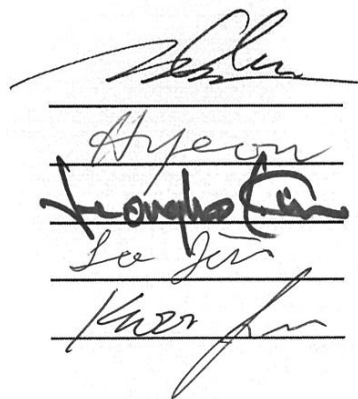
Professor Man CHO, Supervisor

Professor Hyeon PARK

Professor Jeong-Ho KIM

Professor Jinsoo LEE

Professor Kwon JUNG

The image shows five handwritten signatures, each written over a horizontal line. From top to bottom, the signatures are: 1. A signature that appears to be 'Man Cho'. 2. A signature that appears to be 'Hyeon Park'. 3. A signature that appears to be 'Jeong-Ho Kim'. 4. A signature that appears to be 'Jinsoo Lee'. 5. A signature that appears to be 'Kwon Jung'.

Approval as of December, 2015

ABSTRACT

THE OPTIMAL RISK PREMIUM OF BTL(BUILD-TRANSFER-LEASE) PROJECT: THEORETICAL AND EMPIRICAL INVESTIGATION

By

Soojin Park

Korea has initiated total 435 BTL projects from 2005 to 2013, and the importance of BTL type PPP is still increasing. As the number of PPP projects accumulates, public criticism that private investors exploit too high profit from infrastructure also increases. From the policy point of view, it is a critical issue whether the average contract return is proper level in proportion to the investment risk. The objective of this paper is to estimate the optimal level of average risk spread for BTL projects, and test whether the BTL market of Korea is efficient. The study divides BTL investment period into construction and operation phases and applies CAPM to the former and Merton's option model to the latter part to estimate the optimal level of risk spread. Total 426 BTL projects are stratified into four sectors (environment, military, education, culture) and the optimal spreads are measured in each of sectors. The main results show: the optimal BTL risk spreads are estimated to be between 0.86% and 0.95% depending on the sectors, whereas the range of historical contract average is from 1.03% to 1.30%. The hypothesis test concludes that the average contract risk spread (alpha) of BTL in Korea is higher than the optimal level under 99% of confidence level. The result may indicate BTL market of Korea is not efficient. Empirical analysis is additionally performed to identify what idiosyncratic factors influenced how much impact on the historical contract returns. The result

provides supporting evidence of the different levels of idiosyncratic risks among sectors. The funding cost spread is proved to have the strongest impact on the BTL return, and the degree of market competition also makes considerable impact. But the influence of project size is small, meaning BTL investors do not require increased marginal return for bigger investment scale. The result also shows the gaps of average risk spreads among sectors are bigger when measured from regression line than those measured from optimal risk spreads. It may imply the investors in the education and culture sectors require higher excess returns than those in environment sector, implicitly recognizing additional idiosyncratic risks. The analysis also reveals that the return of BTL has some degree of rigidity against the change of market rate, meaning BTL investors look for a long-term return and thus, temporary decrease of market rate makes them require higher risk spread to achieve target return. Out of the above findings, several policy implications are discussed for the further improvement of PPP system: rigorous ex-ante project assessment, inducement of market competition, capable private partner selection, and optimal risk sharing.

ACKNOWLEDGMENT

It is a genuine pleasure to express my deepest sense of thanks and gratitude to my supervisor Dr. Man Cho, and my advisors Dr. Hyeon Park, Dr. Jungho Kim, Dr. Jinsu Lee, and Dr. Kwon Jung.

My mentor and honorable philosopher Dr. Man Cho. His dedication and keen interest to help me had been solely and mainly responsible for the completion of my work. His timely advice, meticulous scrutiny, scholarly approach have helped me, to a very great extent, in accomplishing this task.

I owe a deepest sense of gratitude to Dr. Kwon Jung and Dr. Hyeon Park not only for leading me to the KDI school, but also for the awesome advices that enabled me to complete my courses. And it is my privilege to thank Dr. Jungho Kim, who is a pioneer and great authority on PPP of Korea, for inspiring me of the thesis. I am also extremely thankful to Dr. Jinsu Lee for nice encouragement and great suggestions during my research pursuit.

My beloved families, Chris, Dan and Debbie. I appreciate all of your supports and virtue of patience throughout the PhD program.

And, my Mom and Dad, who always pray for me. I owe all that I achieved to you. Thank you so much.

TABLE OF CONTENTS

I. Introduction	1
II. Overview of BTL in Korea	6
III. Literature Review and Research Design	11
1. Literature review	11
2. Research Design	17
IV. The optimal risk premium for BTL projects	23
1. The optimal return in construction period ($r_{c,t}^*$)	23
2. The optimal return in operation period ($r_{o,t}^*$)	37
(1) Merton's option theory and BTL risk premium in operation period	37
(2) Estimation of the optimal return ($r_{o,t}^*$) in operation period	43
3. The optimal return (r_t^*) and risk spread (α^*) of BTL projects	54
4. Hypothesis test	57
V. Idiosyncratic determinants of BTL project return	60
1. Historical profit level of BTL contracts	61
2. Empirical analysis	64
(1) Financial market	68
(2) Project characteristics	69
(3) Procuring process and Market maturity	73
VI. Policy implication	75
(1) Rigorous project assessment	75
(2) Market competition and maturity	76
(3) Proper partner selection and risk sharing	78
VII. Conclusion	80
REFERENCE	84

APPENDIX	88
(1) Construction sponsor data statistics for beta calculation	88
(2) Equality test among estimated betas	88
(3) Calculation of risk free rate and long-term investment risk premium	89
(4) Calculation of market return	89
(5) SPC selection data statistics for Δ NOI estimation	91
(6) Calculation of risk spread in operation (rs^*_o) period using coupon spread (${}_0cs_t$)	91
(7) Zero mean equality test on expected asset value volatility	92
(8) Θ ratio derivation	93
(9) Empirical test without risk free rate variable	94
(10) The original expression of Merton model and application	94

LIST OF TABLES

Table 1. PPP track record in Korea	9
Table 2. PPP investment ratio to government's infrastructure investment	9
Table 3. Estimated average beta of construction sponsors	28
Table 4. Estimated average beta of construction sponsors in each of facility groups	29
Table 5. Beta revision reflecting difference in financial ratio	32
Table 6. Summary of input and output data to CAPM	34
Table 7. The optimal return during the construction period ($r_{c,t}^*$)	36
Table 8. Regression analysis for NOI estimation	47
Table 9. Summary of the correlation coefficients, used for NOI estimation	49
Table 10. Unexpected volatility of asset value	50
Table 11. Assumptions for estimating risk spread of operation period	51
Table 12. Estimated coupon spread with maturity 't' (${}_0cs_t$)	51
Table 13. Optimal BTL risk spread in operation periods ($rs_{o,t}^*$)	52
Table 14. Optimal BTL risk spread ($rs_{o,t}^*$) and return ($r_{o,t}^*$) in operation period	53
Table 15. Optimal BTL return for the life cycle project period	54
Table 16. Optimal BTL spread (α^*) and average $\alpha_{contract}$	54
Table 17. Historical alpha and optimal level estimation result	57
Table 18. The hypothesis test result	58
Table 19. Project return and alpha statistics	63
Table 20. Candidate determinants of risk spread (alpha)	64
Table 21. Regression result of alpha on determinants	66
Table 22. Difference of average risk spread above environment sector	71
Table 23. Distribution of estimation errors in four sectors	72
Table 24. Gap of optimal alpha among sectors	73

LIST OF FIGURES

Figure 1.	The return of BTL project	2
Figure 2.	PPP implementation structure	6
Figure 3.	PPP track record of Korea	8
Figure 4.	Track record of BTL in Korea	10
Figure 5.	Risk profile of BTL investment	18
Figure 6.	Relationship between ROI and WACC measurement	24
Figure 7.	Bond holder's pay-off position at maturity	38
Figure 8.	SPC's cash flow in BTL investment	40
Figure 9.	Estimation of BTL risk spread during operation period ($r^*_{o,t}$)	43
Figure 10-1.	NOI estimation line (env)	48
Figure 10-2.	NOI estimation line (mil)	48
Figure 10-3.	NOI estimation line (edu)	48
Figure 10-4.	NOI estimation line (cult)	48
Figure 11.	Distributions of unexpected volatility of asset values	50
Figure 12.	Contract average and optimal level of risk spread (α^*)	55
Figure 13.	Time series trend of contract alpha	62
Figure 14.	Distribution of historical contract alpha	62

I. Introduction

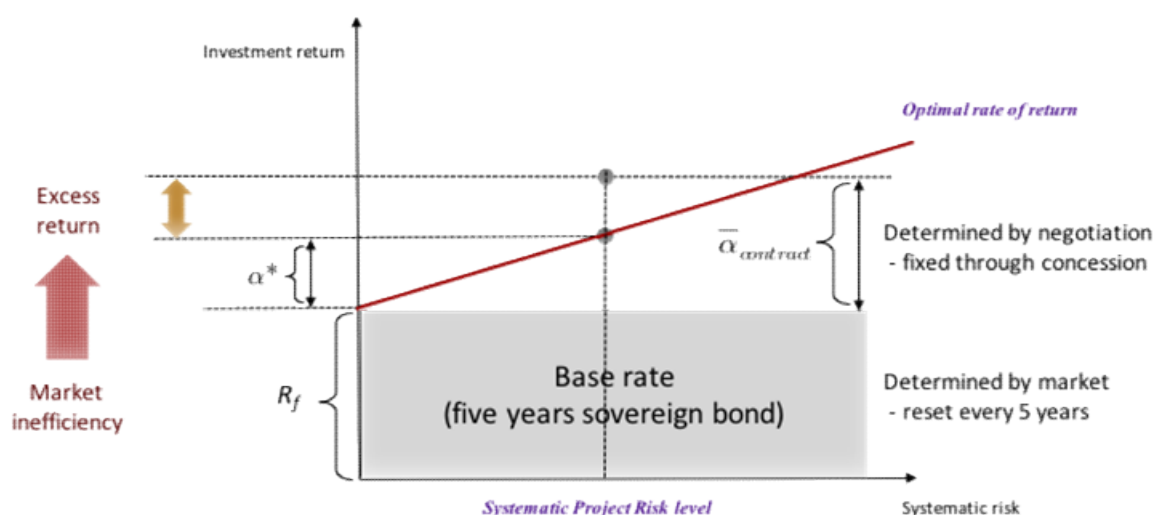
If PPP is properly structured to let the participants pursue well designed goal, the efficiency of the infrastructure services will be able to be enhanced. The Improved efficiency of services and operations can increase the chance that those services are economically sustainable and provided at affordable rate. PPP allows governments to pass part of roles to efficient private sector partners while retaining focus on the core public sector responsibilities such as comprehensive planning, regulation, and supervision. Properly implemented, this approach can result in a lower aggregate cash outlay for a government with better services to tax payers than traditional procurement. To achieve these benefits of PPP, it is a precondition that the granted return to private party should be in affordable level.

During PPP project implementation process, most of the governments are worried whether the profit level of PPP contracts, granted to private investors, are properly determined in proportion to the level of risk that private investors shoulder. Without exception, the comparatively high level of user charge in PPP facility has caused a strong public criticism that private investors exploit too much excess return through monopolistic nature of infrastructure business in Korea. There may be lots of reasons for excess return in infrastructure business, if it exists. One of the probable candidate is the market inefficiency, most of which is caused by idiosyncratic factors. Numerous papers maintain that infrastructure markets are not efficient and thus, excess returns should exist. Vecchi V et al. (2013) suggested that the average of realized excess equity return over the estimated WACC is 9.27% in the healthcare Private Finance Initiative (PFI) contracts of UK. The result confirms that the sponsors who joined the analyzed projects have extracted higher returns than their cost of capital. Regan., et al, (2011), Sawant (2010) maintained that infrastructure market is lowly competitive due to high barriers to entry. Bird et. al., (2012) examined the behavior of infrastructure market returns, and found infrastructure indices exhibit excess returns

with low level of systematic risk. Rothballer., et al. (2012) argued that the reason for the low systematic risk is due to the lower level of market competition in infrastructure industry due to the high level of fixed capital investment requirements. Newbery (2002); Finkenzeller et. al. (2010) argued that many infrastructure investments operate in oligopolistic or nearly monopolistic markets, which structure may explain the low systematic risk identified in infrastructure returns.

If these study results are applicable to Korean BTL¹⁾ market, the public opinion that PPP investors get excess return may be true, and there may exist some policy improvement points in the PPP system of Korea. Accordingly, it becomes a key question whether inefficiency exists in the BTL market of Korea and what are the determining factors that cause the excess return. The purpose of this paper is to estimate optimal average risk spread of BTL projects and compare it with the average contract record to determine whether there exists inefficiency in the BTL market of Korea. Additionally, empirical analysis is performed to find out idiosyncratic factors that influenced on the contract returns. The paper concludes with summary of findings and policy implications for the further improvement of PPP system.

Figure 1. The return of BTL project



1) BLT(Build-Lease-Transfer) contract is more common term than BTL(Build-Transfer-Lease) contract that is unique in Korea. The difference is timing of property ownership transfer. Korean government prefers to own property right of public infrastructures as soon as they are built. In return for the property ownership, the procuring authority grants a 'concession right' that designates the right of generating profit through the facility and responsibility for maintaining the property during the concession period. Private investors recognize the 'concession right' as an intangible asset in their balance sheet and amortize through concession period.

As is illustrated in above figure, the red line represents the optimal level of BTL investment return, corresponding to systematic risk of project. The total return of BTL is comprised of two parts: base rate (five years sovereign bond rate) and risk spread, which is, so called, ‘alpha’ in the Standard Concession Agreements of Korea. The former is determined by macro market and the latter is determined through bidding and negotiation as a compensation for investment risk. As a result, the optimal rate of alpha should reflect only systematic market risk that cannot be removed any more through the diversification of portfolio. Conceptually, the optimal level of risk spread (α^*) is similar to a market premium multiplied by beta²⁾ in conventional CAPM theory if we consider the base rate as a proxy to risk free rate. In general, the optimal level of alpha supposed to be lower than the average alpha of historical contracts because the infrastructure market is known as inefficient. Some idiosyncratic factors may have affected to the level of alpha in contract. However, the higher level of contract alpha does not necessarily mean a mispricing because there should exist some limits in removing unsystematic risk in the real world, especially more limits exist in infrastructure market as is mentioned. As a result, the optimal risk spread (α^*) is the estimated risk spread just assuming systematic risk, which does not consider the remaining inefficiency of the real infrastructure market. But the remaining market inefficiency may be reflected to required rate of return.

If we could understand what risk factors influence on the required investment return of BTL and measure them in quantitative manner, it would provide a practical framework for pricing corresponding project risk. From the public point of view, it would help preventing both errors of accepting inappropriate projects or rejecting eligible ones, eventually resulting in optimized resource allocation and broader range of infrastructure investment. At the same time, private investors may be able to use the framework as a reference to set their target investment return pertaining to specific project. The Public Private Partnership Handbook of Asian Development Bank summarizes public sector’s motivations for involving Public Private Partnership (PPP):

2) $[E(R_m) - R_f] \times \beta$

mobilization of private capital, tool for better efficiency, and catalyst for broader reform. Governments face an everlasting insufficiency in funding infrastructures required to support growing population and GDP. They are also challenged by increasing urbanization, rehabilitation of aged infrastructures, needs for expanding networks to new cities, and the goal of reaching under-served area. Furthermore, infrastructure services are often provided under operating cost level, which is covered only through government's budget, and thus causing an additional drain from public resources. Combined with limited financial capacity, this pressure drives a desire to mobilize private capital for infrastructure investment. If correctly structured, PPP may be able to mobilize previously untapped resources from local or international private sector. Although the efficient use of scarce public resources should be a critical challenge for governments, many of them fall far short of that goal. The reason is public sector typically has few incentives for efficiency in its organization and processes. Injecting a motivation into an entrenched public sector is difficult, though not impossible. Private sector operators, in contrast, enter into an investment or contracting opportunity with the clear goal of maximizing profit, which is generated by innovation and efficiency.

Although many researches have studied the taxonomy of PPP risks and their allocation between private investors and government from policy point of view, there are not many papers that researched on the optimal level of return for BTL. The complexity caused from mixed knowledge of construction, operation, law, and finance may make it hard to study PPP from a single view of academic expertise. The harder obstacle is an access to meaningful project data covering construction, operation, and concession agreement clauses. There are not even many economies that have meaningful number of contract record in BTL. Even though there exist several researches on proper PPP rate of return, most of the conventional researches are focused on Build-Transfer-Operation(BTO) because BTL was introduced comparatively late in Korea³⁾. And further, project information is sporadically managed by each

3) The BTO type commenced with the enactment of PPP law in 1994, and BTL was introduced in 2005 in Korea

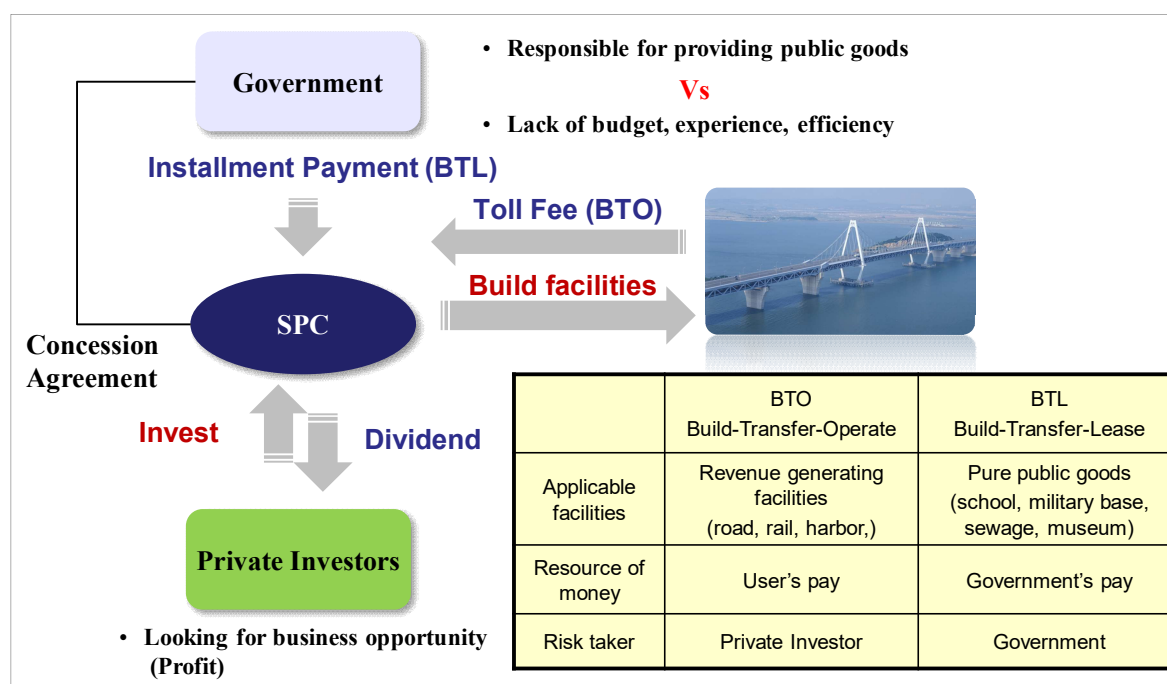
department, ministry, or municipal governments. All of these factors may have hindered researchers from making quantitative risk analysis based on financial data of BTL.

The study begins with ‘II. Overview of BTL in Korea’, which introduces the business structure and risk profile of BTL contract. It also explains the BTL market status in Korea. The following chapter, ‘III. Literature Review and Research Design’ introduces relevant studies, new framework of this research, and the applied methodologies. The chapter ‘IV. The Optimal Risk Premium for BTL projects’ estimates the optimal level of BTL return and tests whether the historical average contract return is not different from the estimated optimal level. Some idiosyncratic factors that may have affected on the average contract spread are further investigated in the following chapter ‘V. Idiosyncratic determinants of BTL project return’. The chapter analyzes what idiosyncratic factors have how much influence on the average contract risk spread of BTL on top of systematic risk. On the basis of hypothesis test and empirical analysis, the study suggests several political implications for further improvement of PPP policy in chapter ‘VI. Policy implication’. And finally, it closes with the chapter ‘VII. Conclusion’.

II. Overview of BTL in Korea

There exist various forms of PPP contracts around the world. Among them, leasing and concession types are popular. The Build-Transfer-Lease (BTL) contract is one of the representative form of leasing which is similar to the availability payment concept.

Figure 2. PPP implementation structure



(Source: PIMAC training material)

On the basis of a concession agreement, private investor builds a facility, which ownership is transferred to procuring government when construction is completed. In return for the ownership transfer, procuring government issues a certificate that designates a right of receiving installment payment and responsibility of operation. Government's payment to private investor is comprised of pre-determined budget operation cost and installment payment that is a compensation for the initial capital cost plus profit margin. Examples of BTL type projects are school, military base, sewage system, museum, library, community centers. In other way round, a BTO

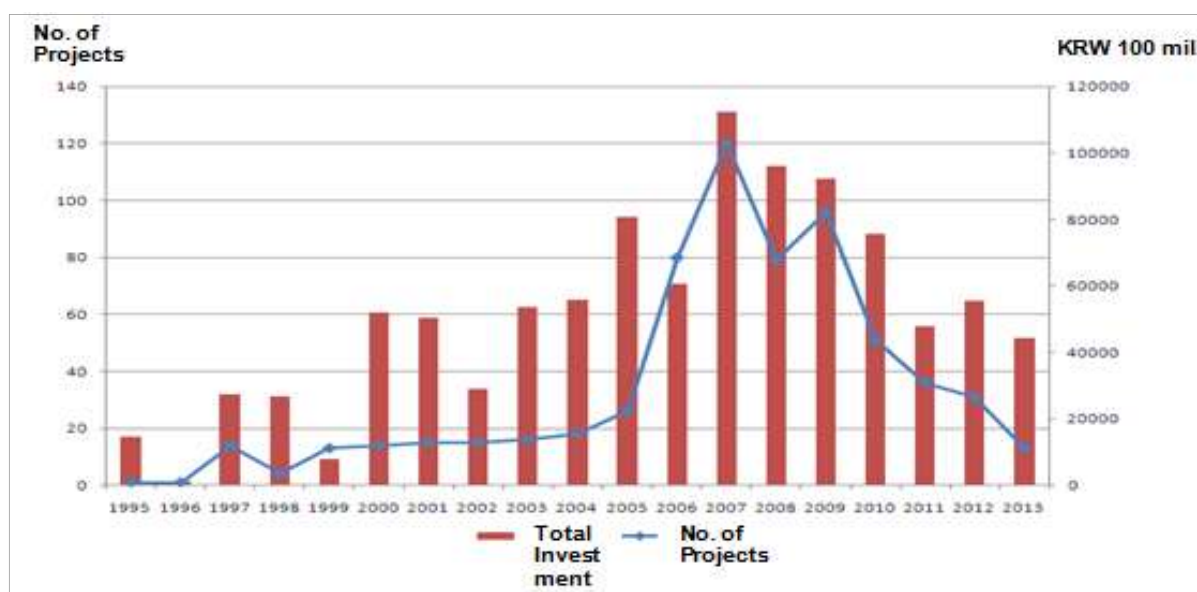
contract gives a private concessionaire full responsibility for the delivery of services, including construction, operation, fee collection, and maintenance of the facility. The key difference from BTL is that the concessionaire collects tariff directly from users to fulfill its target profit, and thus a private investor bears the demand risk. So called, economic infrastructures that generate cash are built through BTO in Korea. Examples are road, train, harbor, parking lot etc.

'The Promotion of Private Capital into Social Overhead Capital Investment Act' was introduced to Korea in 1994 as a way of supplementing government finances and building infrastructure with private funding. On this legal basis, solicited projects for the construction and operation of economic infrastructures, such as road, railway, and port, began to proceed under the build-transfer-operation (BTO) scheme. This Act was later modified to become the 'Act on Private Participation in Infrastructure' in December 1999. The focus of the changes was to encourage PPP as a way of overcoming the Asian financial Crisis that had affected Korea from 1997. The Act adopted unsolicited projects as a new form of PPP. In January 2005, the 'Act on Private Participation in Infrastructure' was further amended into the 'Act on Public-Private Partnerships in Infrastructure (PPP Act)'. The eligible PPP type was expanded with the introduction of the build-transfer-lease (BTL) framework for social infrastructures such as school and military housing facility. A system of pre-project assessment was adopted. Based on the assessment report, the aggregate investment ceiling for BTL and (prospective) investment ceilings for target facilities should be submitted to the National Assembly with the budget for the following accounting year for approval. The legal system for public-private partnerships currently consists of PPP Act, associated Enforcement Decree, and Basic Plan. The Act has elements of both a general law that comprehensively prescribes typical details regarding PPP projects such as eligible facilities, implementation methods and procedures, government's support, management, oversight, and restriction measures as well as a special law that precedes other individual laws. In accordance with the PPP Act and its associated Enforcement Decree, the Ministry of Strategy and Finance drafts and

distributes the Basic Plan for PPP, specifying the detailed procedures and methods needed for PPP project execution. The Basic Plan for PPP serves as guidelines for PPP projects alongside the Act and Enforcement Decree. The plan prescribes procedures and general rules for PPP projects.

Ever since the PPP act was legislated, the use of PPP has increased in Korea due to the needs for building infrastructure to support rapid economic growth in the country. The mobilized private capital injection has played a crucial role for the equipment of backbone infrastructures by supplementing the shortfall of public funds. The number of projects and contract amount rose steady after the introduction of unsolicited projects and MRGs in 1999. The number of projects went through another rapid increase after the introduction of the BTL scheme in 2005.

Figure 3. PPP track record of Korea



(Source: Ministry of Strategy and Finance of Korea)

Total 651 projects with investment value of \$88.53 billion were either being operated or signed as of December 2013 in Korea. Among which, the BTO type is applied to 216 projects with the value of approximately \$63.76 billion and the BTL type is applied to 435 projects, which value amounts about \$24.77 billion⁴).

4) Lee, H., Park, S., 2014, 'Success Stories and Lessons Learned from Public-Private Partnership Projects in Korea', KDI

Table 1. PPP track record in Korea

(Units: bill USD, %)

Category	No. of Projects	Percentage	Total Invested	Percentage
BTO	216	33%	63.76	72%
BTL	435	77%	24.77	28%
Total	651	100%	88.53	100.00%

(Source: Ministry of Strategy and Finance, 2014)

If the PPP had not been introduced, the substantial investments would have not been able to made by the public sector alone, and the failure to implement projects because of this financial burden would have resulted in an equivalent loss in terms of economic growth and convenience of the public.

Table 2. PPP investment ratio to government's infrastructure investment

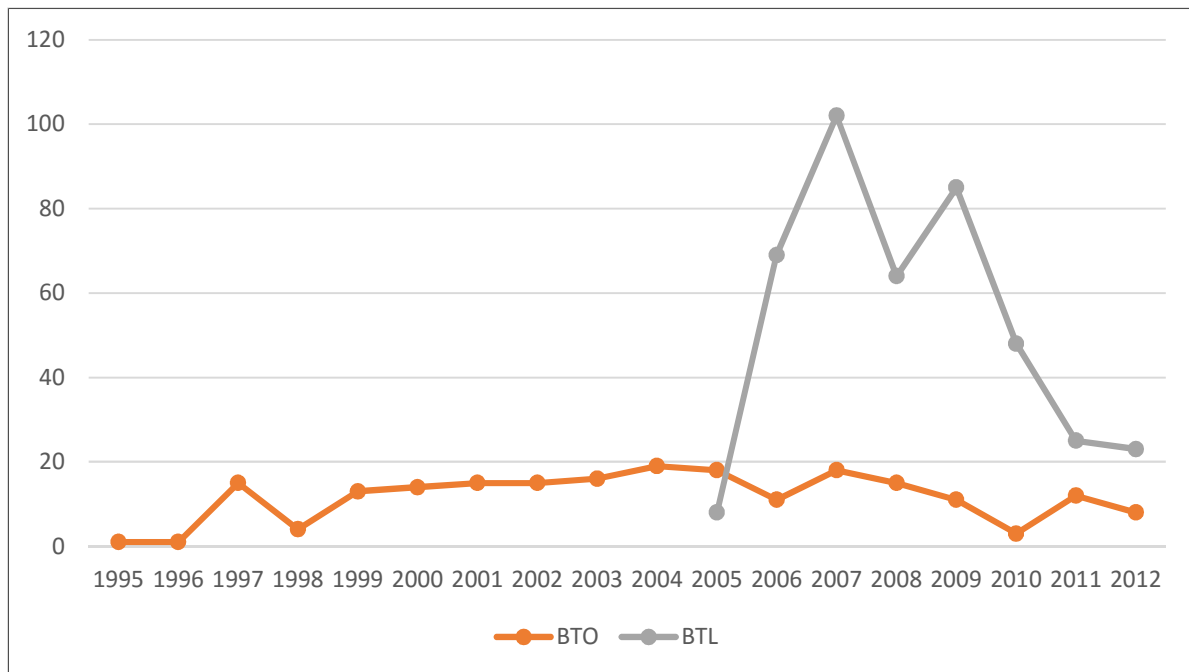
(Trill KRW, %)

Funding Source	'98	'00	'01	'02	'03	'04	'05	'06	'07	'08	'09	'10	'11
Private Investment (A)	0.5	1.0	0.6	1.2	1.0	1.7	2.9	2.9	3.1	3.8	3.9	2.7	2.2
Gov't Investment (B)	12.7	15.2	16.0	16.0	18.4	17.4	18.3	18.4	18.4	20.5	25.4	25.1	24.4
A / B (%)	3.9	6.6	3.4	7.5	5.6	9.8	16.1	15.9	17.0	18.4	15.4	11.0	9.0

(Source: Ministry of Strategy and Finance)

The Standard Concession Agreement of Korea suggests that the contract return of BTL is comprised of risk spread (alpha) plus base rate, for which five years variable sovereign bond rate is used. The meaningful milestone that really determines the level of return for BTL is the risk spread (alpha) because the base rate changes. The risk spread (alpha), which can't be changed throughout contract period, is determined through bidding and negotiation. The average alpha of 462 BTL contracts from 1995 to 2014 is 1.13%. It ranges from 0.9% to 1.3% and did not show a time-series trend with flat shape. Recently in 2014, it has dropped down to 1% range due to the depressed interest rate and hard PPP market competition in Korea.

Figure 4. Track record of BTL in Korea



(Source: Ministry of Strategy and Finance)

Recently, the importance of BTL type has been increasing in Korea. As the economic status get matured, interests in social infrastructures such as school, hospital, culture facilities have surged, while that for economic infrastructures such as road, train has been diminishing. Other developing economies will experience a similar trend as PPP market and economic development status get matured, resulting in more request for study on BTL return. As is illustrated in the above figure, the history of BTL is comparatively shorter than that of BTO in Korea, as a result, the study on BTL return is hard to find out. This study may contribute to academia with a new study topic and suggestion of analytical framework, and can also help both practitioners from public and private in determining mutually agreeable range of BTL return.

III. Literature Review and Research Design

1. Literature review

Although scarce in numbers, there are several researches that tried to estimate optimal level of PPP return based on the Capital Asset Pricing Model (CAPM). One of the latest work is Vecchi V et al. (2013)⁵⁾. The study takes account of the corporate hurdle rate approach which is the dominant capital budgeting practice among equity investors in Private Finance Initiative (PFI). The study seeks to judge whether excess equity returns exist although this method, the author mentions, may over-estimate the appropriate risk premium in comparison with a benchmark related to the systematic risk associated with PFI projects. Using the 77 samples of healthcare PFI contracts from 1997 to 2011 in UK, the study measured Weighted Average Cost of Capital (WACC) of sponsors as a proxy for optimal equity IRR. The average of realized excess equity return over the estimated WACC is 9.27%. The result confirms that the sponsors who joined the analyzed projects have extracted higher returns than their cost of capital. The authors added the opinion that the most likely source of excess return is a lack of competition in the market for contracts, therefore eliminating complexity in procurement should be a key focus of policy. The study used the beta of sponsors to estimate WACC of PFI projects. However, the key concept of PPP is to allocate risks to the participants who can best handle, and thus the remaining systematic risk of private investor is decreased as much as allocated to public. For this reason, conceptually, the optimal return of private investor in PPP will be smaller than what is estimated based on the benchmarking beta of sponsors or similar industries.

Bird et al., (2012) measured a proper level of infrastructure investment return, using Fama-French three factor model. The study used UBS infrastructure index,

5) Vecchi V, et al., 2013, 'Does the private sector receive an excess return from investment in health care infrastructure projects? Evidence from the UK', Health Policy

which included 200 infrastructure stocks. The index comprises 40% of stocks from US, 40% from Europe, and remaining 20% from Asia. The study estimated beta of listed infrastructure and utility companies in United States from 1995 to 2009, resulting in 0.49. Whereas, the utility beta from all three infrastructure markets is estimated as 0.57. The beta of listed infrastructure and utility companies in Australia was estimated as 0.63, but when unlisted companies were included, the estimated beta dropped to 0.6. This result implies that the stock value of unlisted companies are comparatively stable and overall infrastructure indices exhibit excess returns with low levels of systematic risks.

There are also series of studies focused on PPP returns in Korea. Park, J., et al. (2006) studied a model for determining optimal risk premium for BTL project risk. The study assumed project return and equity ratio can be determined under several limits such as $NPV \geq 0$, $ROE \geq ROE_{min}$, and $DSCR \geq DSCR_{min}$. The minimum level of ROE is assumed 8.0%, and that of Debt Service Coverage Ratio (DSCR) is 1.30. The study applied Monte Carlo Simulation algorithm model with constant inputs and risk variables such as construction cost, operation cost, inflation rate, and risk free rate volatility. The estimated result of risk spread for BTL is 2.1% under 21.54% of equity ratio, 8.23% of ROE and 1.3X of DSCR. The Study is meaningful in that it suggested a quantitative tool for estimating the risk spread that make a project financially feasible. However, the topic of above research is a bit different from this study in that above research did not focused on the ‘market price’ of risk spread but technically ‘feasible spread’ that investors should achieve under given target of ROE_{min} and $DSCR_{min}$. And that, the major risk variables such as distribution of construction cost, operation cost, and interest rates are not observed data⁶⁾ but discretionary inputs based on interview with industry experts. So, the study result and approach can’t be directly applied to estimating optimal market price of risk spread, and testing market inefficiency in BTL.

6) It is understandable because the BTL was introduced in 2006 and thus, there was no track record for these variables. The study used 10%, 50% and 90% of distribution of volatility with 25%, 50%, and 25% of probabilities are assigned

Jang, et al. (2006) also tried to estimate proper rate of return on BTL. The study assumed that investment risk spread of BTL can be measured by insurance premium plus long-term investment premium. They used a project data (Haman-Jinju link) to measure insurance premium, which is 0.36%. They added 0.53% of long term premium on top of the above insurance premium, resulting in 0.89% of risk spread for the sample project. The research has flaws in two points. First of all, an insurance premium can not reflect all business risk because there is no insurance that can transfer the whole business risk. If all business risk is transferred, it means ownership of equity is transferred. That is why there still remains dividend paid to equity holders after paying insurance premium. Second, the study measured insurance premium based on one single project, which can not be generalized.

Shim, et al. (2006) researched cost of equity capital for BTO in Korea based on the financial information of special purpose company (SPC), using CAPM. This study is one of the earliest pioneering studies that tried to estimate cost of capital for PPP project in Korea.

Park (2008) measured the optimal level of return for private investors of BTO projects in Korea and additionally surveyed the properness of risk premium.

Choi, et al. (2013)⁷⁾ researched the required rate of return for road BTO projects of Korea. The paper studied optimal level of return in time series, reflecting changes of macro-economic factors. It suggested that average historical beta of road BTO of Korea is 0.89. On the basis of the beta, optimal level of after-tax nominal average return for BTO from 1999 to 2005 is estimated to be 13.2%, while the estimated average return from 2006 to 2011 is 9.47%. The study applied the weighted average beta of shareholders as a proxy for BTO business for the whole life-cycle investment including operation period. So, the study result represents the WACC of sponsors not specifically that of BTO business itself.

7) Choi, J., Park, T., 2013, 'A study in the Reasonable Rate of Return for the Korean PPI Projects: An Investigation on Transportation Projects', The Seoul Institute

Shin (2009)⁸⁾ assumed that investment return of BTO is comprised of liquidity premium, construction risk premium, operation risk premium, and demand forecast risk premium. The study also considered option values of termination clause both by government and private investor to estimate optimal level of investor's return. The estimated range of optimal BTO return is from 6.61% to 8.33%. This study is meaningful in that it realized diverse nature of systematic risk in PPP business. However, this approach can't be directly applied to the study on optimal BTL return because the operation risk of BTL is totally different from BTO. For example, not BTL investors but procuring government bears demand risk.

In summary, there are two main issues in applying the above approaches to the study on the optimal return for BTL. First of all, most of the Korean BTL projects are facilities that traditionally government had been operating, such as military bases, sewage systems, or schools. For this reason, there are few listed sponsors that had operated similar kinds of facilities to BTL in Korea. And thus, it is almost impossible to find benchmarking market beta. Some of the conventional researches on proper return of BTO used construction industry beta. However, the BTL in operation period has totally heterogeneous nature of business that is different from construction industry⁹⁾. The second and more critical problem is the changed nature of systematic risk in BTL. The core concept of PPP is a proper risk allocation to the best bearable parties, reducing total managed risk as well as required rate of return. As a result, the portion of systematic risk that BTL investors shoulder should be smaller than the original project risk. For this reason, conceptually, the optimal BTL return of private investor should be smaller than what is estimated based on the benchmarking beta of sponsors or similar industries. Most of the conventional studies didn't consider this reduction of systematic risk in PPP. In summary, the corporate hurdle rate approach based on CAPM may be appropriate for measuring the optimal return of BTO or that

8) Shin, S., 2009, 'A study on the Fair Returns of Private Participants' Investment', Korea Journal of Construction Engineering and Management

9) Even though this is a flaw, it is plausible to measure the systematic risk of BTL in construction period using proxy beta of construction sponsors

of BTL under construction period because most of the life-cycle risk is on the hand of private investor in BTO concession¹⁰⁾, and the construction risk is fully on the hand of private investor even in BTL. However, above approach is probably not applicable to estimating BTL return, especially in operation period, because a major portion of systematic risk (i.e. demand risk) is allocated to public party and thus, it has high probability of over-estimation.

Bianchi J. et al. (2014)¹¹⁾ suggested that asset pricing models exhibited poor out-of-sample predictive performance when compared to simple, fixed excess return models for the period 1997 through 2012 in Australia. They employed the same approach as Simin (2008) and Giacomini (2006)¹²⁾ to evaluate the predictive performance of asset pricing models on Australian Securities Exchange (ASX) listed infrastructure and PPP returns. The study used 16 years of monthly return data of ASX from 1997 to 2012. There are two main conclusions in the paper: First of all, 9~10% of fixed excess return model showed the best predictor of infrastructure returns across the entire analysis period, which result is also consistent with that of Simin (2008) in US settings, and that of Whittaker (2013) in the Australian settings. Second, both the conventional asset pricing and fixed excess return models are generally over-estimating future two year returns, on average. The paper provides empirical evidence that CAPM tends to over-estimate return on PPP, and is not appropriate model in estimating optimal return for PPP projects, especially over a long term horizon.

The poor predictive performance of CAPM may be caused by two main reasons. As is discussed, considerable portion of the PPP business risk in operation is shared by public and thus, the nature of investor's risk is totally changed into similar form to bond (especially in BTL case) rather than that of stock. Second reason may come from the complex and mixed nature of PPP business. PPP concessions are comprised

10) Of course, government share part of business risk in BTO, too. But the degree of allocated systematic risk to private investor is far higher than that of BTL.

11) Robert, J., Michael, E., and Timothy W., 2014, 'The predictability of Australian Listed Infrastructure and Public-Private Partnership Returns Using Asset Pricing Models.' CSIRO-Monash Superannuation Research Cluster

12) They used the Root Mean Square Forecast Error (RMSFE) as a measure of forecast accuracy

of planning, financing, construction and operation, all of which are totally different characters of businesses. For this reason, it is almost impossible to find out one representative benchmarking industry beta that can explain the systematic risk of the whole life-cycle PPP business. Moreover, the demand risk of BTL is shouldered by government during operation period because government pays revenue to SPC in a similar form to installment payment, which is totally different from the nature of equity investment.

This research suggests a new framework in estimating the optimal level of market return in BTL, which was not previously studied enough. As is mentioned, there are not many papers that researched on the optimal level of return, specifically in BTL, mainly due to the lack of empirical data. This research uses 426 project data that cover construction, operation, and financing clauses. This research also applies a multi-dimensional approach (instead of a single CAPM), dividing investment duration into construction and operation periods based on the different business natures in respective phases. Especially, a credit risk spread measurement approach of bond is applied in estimating the operation risk of BTL business. This research also tries to measure the optimal average BTL return at market level, not single project level, stratifying facility sectors. Additionally, idiosyncratic factors are analyzed to explain the difference between optimal returns and average contract returns. All of these efforts may differentiate this research from previous studies. And the result may help procuring government in assessing the efficacy of using the BTL in policy setting, and may provide private investors with a reference to proper return for BTL investment.

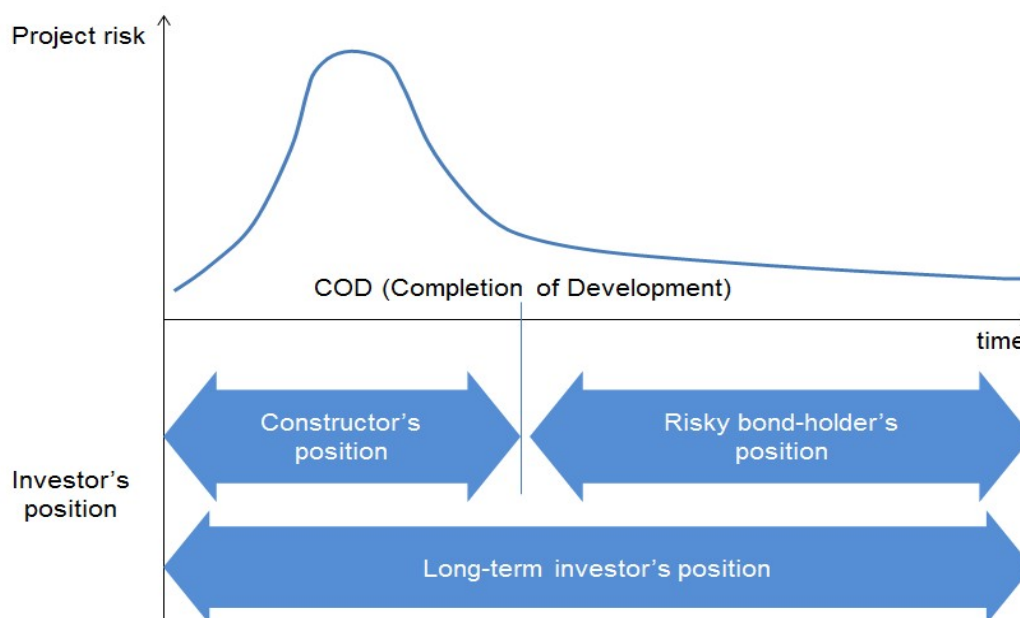
2. Research Design

There are four major business risks¹³⁾ in PPP investment: construction, demand, operation, and long-term investment risks. The demand risk occurs from the uncertainty of actual demand. In general, the revenue of BTO projects depends on quantity of users at pre-determined price level. If the actual quantity of user is lower than the forecasted, the possibility of default will increase because net operation income may not be able to cover debt service amount. However, in BTL type contract, government pays pre-determined revenue to project company regardless of usage, and thus a project company doesn't bear demand risk. During the project planning phase, procuring authority decides to purchase, so called, social facilities that should be built regardless of demand level. In most cases, those facilities are basic social services that government must provide such as sewage system, school, military base or culture facilities. The construction risk generally has critical impact on the success of PPP projects because construction cost takes the biggest portion of the invested capital. If a cost overrun or schedule delay occurs during construction period, a project may even go default. Another critical risk in PPP investment is operation risk. From the financial point of view, the operation risk may be said as 'any uncertainty that can affect the net operation income ('NOI' hereafter)' during operation period. The degree of operation risk can be measured by deviation of realized NOI from the forecasted. The long-term investment risk exists in PPP because concession periods are usually over 20 years during which period interest rate change or liquidity shortfall may happen. In return for the long-term investment risk, sponsors require some level of term-premium. In summary, BTL investors do not bear demand risk but still hold construction, operation, and long-term investment risks. These BTL investor's risks can be divided depending on chronological sequence. In developing phase, a BTL investor has a similar position to construction company, but during operation period, it has a similar position to a bond-holder, whose cash flow is affected by the

13) Apart from business risk, many other categories of risks such as environmental, political, reputation, force-majeure, etc. This research assumes other categories of risks are same in all projects because the research scope is limited to local BTL

volatility of operation cost plus other variables. As a result, a life-cycle BTL investor's position is similar to buying a long-term sovereign bond that additionally possesses construction and operation risk.

Figure 5. Risk profile of BTL investment



It may be plausible to apply conventional approach of measuring Weighted Average Cost of Capital (WACC) of construction sponsors as a proxy for optimal rate of return in construction phase. However, as is illustrated in above figure, the business characteristics of BTL totally changes after the completion of development (COD). That's why a new approach is applied in measuring the optimal risk premium in this research.

Several reasons may be summarized why a new approach should have been considered instead of the conventional WACC to estimate the optimal return in operation phase. The first reason is the mixed nature of PPP business as is discussed in above paragraph. Most of the BTL concessions include planning, financing, construction and operation, all of which have totally different business characters and they changes in chronicle sequence. It is hard to find out one benchmarking beta that covers all range of business characters through the whole BTL life-cycle. Second, most of the BTL projects are procuring and managing facilities that traditionally

government operated such as military bases, sewage systems, or schools. For this reason, there are few listed private companies that had operated similar sorts of BTL facilities in Korea. And thus, it is almost impossible to find benchmarking market beta for operation phase of BTL. Third, the most critical reason is the changed nature of systematic risk in operation period. The core concept of PPP is a proper risk allocation to the best bearable parties, and by doing so, reduction of total risk as well as required rate of return. For example, the demand risk of BTL is shouldered by government during the operation period because government pays revenue to SPC in a similar form to installment payment, which is totally different from the nature of equity investment. As a result, the remaining systematic risk of BTL investors should be smaller than the original level of business risk, especially in operation phase¹⁴. For this reason, it is a matter of course that the optimal BTL return of private investor should be smaller than what is estimated based on a benchmarking beta of sponsors or similar industry and so, separate methodology should be applied for the operation phase. As is introduced, the study of Bianchi J. et al. (2014) supports this argument with empirical evidence that CAPM tends to over-estimate return on PPP, and is not appropriate model in estimating optimal return for PPP projects, especially over a long term horizon. They concluded that fixed excess return models predict PPP performance better than asset pricing models.

The chapter 'IV. Optimal risk premium for BTL projects' investigates the optimal level of BTL return (r_t^*) and test whether the contract average is not different from the estimated level. As is discussed, the investment life-cycle of BTL may be divided into construction and operation periods. We estimate the optimal return of SPC (Special Purpose Company) by weighted average returns of the two periods ($r_{c,t}^*$, $r_{o,t}^*$), additionally considering a long-term investment premium.

$$r_t^* = \theta \cdot r_{c,t}^* + (1-\theta) \cdot r_{o,t}^* \quad (3.1)$$

14) In general, most of construction risk is allocated to private partner whereas, demand and termination risks are allocated to public partner in operation phase, resulting in reduced level of investment risk for private partner.

- r_t^* : optimal return from a BTL project, assessed at time 't' (beginning of project)
 $r_{c,t}^*$: optimal rate of return for construction period, assessed at time 't'
 $r_{o,t}^*$: optimal rate of rerun for operation period, assessed at time 't'
 Θ : share of return for construction period in total (refer to appendix 8)

The optimal in construction period ($r_{c,t}^*$) is measured by the weighted average cost of capital (WACC)¹⁵⁾ based on the average equity share in total asset (K) of BTL project companies, average cost of debt (k_d), and estimated cost of equity capital $E(k_e)$. For this purpose, the average benchmarking beta of construction sponsors that historically joined BTL is calculated. The beta is revised reflecting the leverage ratio of BTL projects, using Hamada model. The revised beta is applied to CAPM to draw the estimated cost of equity capital $E(k_e)$.

$$r_{c,t}^* = WACC = K \cdot E_t[k_e] + (1-K) \cdot E_t[k_d] (1-\tau) \quad (3.2)$$

$$r_{o,t}^* = E_t[rs_{o,t}^*] + R_f$$

- k_e : cost of equity capital (estimated based on CAPM)
 k_d : cost of debt capital (historical average of BTL contracts)
 τ : corporate tax rate (effective marginal corporate tax rate)
 K : equity share in total asset (historical average of BTL contracts)
 $E_t[\cdot]$: expectation operator, assessed at time 't' based on a best possible information set
 $rs_{o,t}^*$: optimal risk spread in operation period
 R_f : risk free rate

The BTL investor's payoff position during operation period is similar to investing in a risky bond, which value mainly depends on the volatility of operation cost. And the volatility comes from the deviation of actual cost from the contracted budget cost of SPC. The risk spread during operation period ($rs_{o,t}^*$) is measured based on the Merton's option model¹⁶⁾. And the optimal rate of return in operation period ($r_{o,t}^*$) is

15) The long-term investment premium is added on top of estimated WACC considering the investment duration of BTL

16) The Merton's option model is one of the 'distance-to-default theory' that requires only distribution of asset value volatility and leverage ratio information for credit risk measurement, while other theories need large population of empirical default rate data. And, the Merton's model can describe the isomorphic pay-off position of BTL's installment payments in the most

calculated by adding a risk free rate (R_f) to the above calculated optimal risk spread for operation period.¹⁷⁾

The optimal BTL return (r^*_t) minus risk free rate (R_f) makes the optimal risk spread of BTL (α^*), which can be fairly compared with historical average contract risk spread ($\alpha_{contract}$)¹⁸⁾. It can be tested whether, in overall, the average contract BTL spread of Korea is not different from the optimal level. In other words, it can be tested whether the inefficiency in BTL market of Korea exist.

The null hypothesis is $H_o: \bar{\alpha}_{contract} - \alpha^* = 0$, and the alternative one is $H_a: \bar{\alpha}_{contract} - \alpha^* \neq 0$. The null hypothesis means 'the historical average of BTL contract spread (alpha) is statistically not different from the optimal risk spread in Korea'. If the null hypothesis is accepted, we will conclude there is no evidence that BTL market of Korea is inefficient. And the alternative hypothesis means 'there is no evidence that the historical average of BTL contract spread (alpha) is statistically not different from the optimal risk spread in Korea'. If the alternative hypothesis is accepted, we may conclude that there exists inefficiency in BTL market of Korea. The optimal risk spread of BTL (α^*) is the 'estimated' spread, which by nature, may have some degree of measurement error. The statistical confidence interval of 99% is considered in the hypothesis test.

If the alternative hypothesis is accepted, it can be said that the BTL market of Korea is not fully efficient, and idiosyncratic factors are reflected to the contract returns. These factors are investigated in the following chapter 'V. Idiosyncratic determinants on BTL return' to draw political implications. The chapter analyzes what idiosyncratic factors have how much influence on the average contract risk spread of BTL on top of the systematic risk. The historical BTL contract returns are regressed on factors such as project size, number of bidders, facility sectors, etc.

similar way.

17) The long-term investment premium is added on top of the estimated optimal return in operation period considering the investment duration of BTL

18) The total returns can't be compared because the base rate changes depending on macro market

Macroeconomic variables such as risk-free rate, and market risk spread are regressed together for control purpose. Also, dummy variables are assigned to stratify facility sectors. On the basis of hypothesis test as well as empirical analysis, the paper suggests political implication for further improvement of PPP policy in chapter 'VI. Policy implication'.

IV. The optimal risk premium for BTL projects

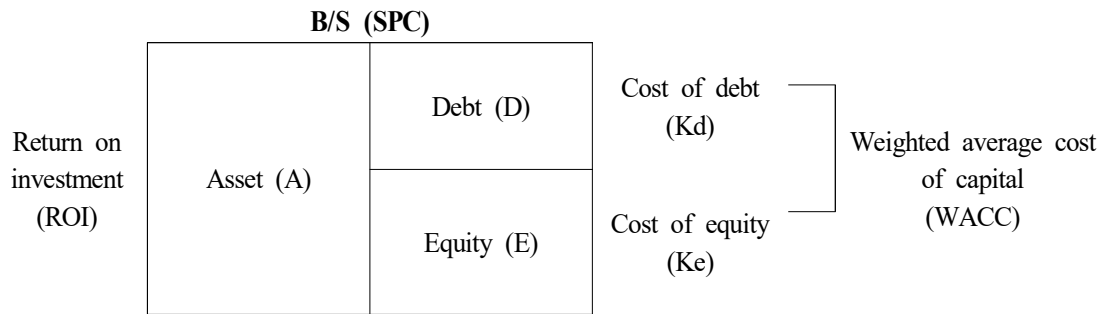
1. The optimal return in construction period ($r_{c,t}^*$)

PPP projects are implemented through SPC (Special Purpose Company), which are generally not listed in Korea and so, hard to get market beta or ROE (return on equity) data. Especially during the construction period, SPC doesn't even generate cash, making it hard to directly measure systematic business risk. So, the optimal rate of return during construction period can't be directly measured by observing operation risk of SPC. As a result, an indirect approach is needed to measure the optimal return of BTL projects during the construction period. The premise used in this paper is that the optimal rate of return during construction period ($r_{c,t}^*$) can be measured using the WACC of SPC, and the return on equity (ROE) of SPC can be estimated by benchmarking the construction sponsor's required rate of return. The estimated WACC as a proxy for optimal return of BTL project may be subject to underestimation because real market is not efficient enough. However, as the purpose of this paper is to measure 'optimal' return, so market noise will not be considered.¹⁹⁾

If a company is rational, it will invest from the best opportunity and choose the next best and so on, until the marginal ROI (return on investment) from the new investment equals to the marginal WACC (Weighted Average Cost of Capital). Assuming a market is efficient, this decision will be made by all market players and thus, the supply of infrastructure will continue until there remains no excess margin. As a result, the weighted average cost of capital shall be equal to the return on investment at the equilibrium point in efficient market.

¹⁹⁾ The effect of idiosyncratic factors will be additionally discussed in the following chapter.

Figure 6. Relationship between ROI and WACC measurement



(Source: Comprehensive PPP evaluation, 2013, Ministry of Strategy and Finance)

Vecchi V, et al. (2013) discussed the rationale of using sponsor's cost of capital as a benchmark to required rate of equity return for SPC in detail: "Our premise is that, in a non-recourse project financing, the lowest acceptable blended equity IRR on sponsors' investment is the sponsors' WACC. Given the relatively low level of systematic risk to which equity returns are exposed (and thus the relatively low beta), the WACC of each sponsor must determine the IRR in a competitive market. It follows that the calculation of the 'fair' equity IRR of a PFI project is based on the calculation of the WACC of each sponsor." This approach is applaudable for measuring construction risk and thus, will be applied in measuring optimal return for BTL projects during construction period. In order for a project to provide an equity return above sponsor's cost of capital, the project IRR must be higher than the weighted average cost of capital of the SPC. The SPC's WACC is, in turn, a function of the cost of the sources of funds used by the SPC: namely, the cost of debt and the cost of equity. Although various kinds of shareholders join BTL project, the construction risk is almost on the shoulder of construction shareholders by contract. Which means the construction risk of BTL project is passed over to construction sponsors. And that, construction sponsors are the best 'comparable industry' in measuring the systematic risk of SPC during construction period because the main business of SPC during construction period is exactly that of construction sponsors'. As a result, we may be able to estimate the SPC's required rate of return

in construction period by measuring that of equity holders responsible for construction.

As is mentioned, the optimal return during the construction period ($r_{c,t}^*$) will be measured by WACC of SPC plus long-term investment premium. The total investment of SPC is comprised of debt and equity and so, the WACC can be estimated by averaging cost of capital based on equity and debt share ratio as below:

$$WACC = K \cdot E_t[k_e] + (1-K) \cdot E_t[k_d] (1-\tau) \quad (4.1)$$

k_e : cost of equity capital (estimated based on CAPM)

k_d : cost of debt capital (observed from historical average of BTL contracts)

τ : corporate tax rate

K : equity share in total asset (observed from historical average of BTL contracts)

$E_t[\cdot]$: expectation operator, assessed at time 't' based on a best possible information set

As most of the shareholders who joined BTL projects in Korea are not listed corporations they don't provide necessary market data to apply revised CAPM such as Fama-French three factor model. The conventional Capital Asset Pricing Model (CAPM) of Sharpe, Lintner and Mossin is used to estimate $E_t[k_e]$. The CAPM states that the cost of equity capital is the sum of risk free rate and a risk premium for the systematic risk of asset. The model can be expressed in slightly revised form in the context of measuring optimal average return of BTL during construction period as follows:

$$E_t[k_e] = R_f + [E(Rm) - R_f] \times \beta_a \quad (4.2)$$

R_f : risk free rate (a return on zero variance asset)

Rm : return on market portfolio

β_a : the average of relative risk in assets to market return during construction period

Determining beta (β_a) will be the hardest part in measuring $E_t[k_e]$. As the goal of this paper is to investigate the average optimal return of BTL projects, we need to

know the average beta (β_a) of all BTL SPCs instead of a beta (β_i) of one specific project. So we would like to constitute an average beta with all construction sponsors that have joined BTL projects in Korea. It's similar to forming a big portfolio by pooling all BTL projects and calculating an average beta (β_a) of BTL projects during construction period using proxy beta of construction sponsors. In most of cases, construction companies join BTL as both an equity investors and EPC (Engineering Procurement and Construction) contractors, responsible for completion of construction. And, as is mentioned, the nature of BTL business during construction period is similar to that of construction sponsors. As a result, average systematic risk (β_a) of construction shareholders that have joined BTL projects will most likely to represent the average beta of all BTL projects²⁰).

Conceptually, the easiest way is to collect all betas of construction companies that have joined BTL projects and simply average them. However, regret to say, most of the construction sponsors of BTL projects are middle or small sized, unlisted companies that don't have market beta (β). So we have no choice but to calculate beta of construction sponsors based on financial statements. Lot's of theories were developed in estimating accounting beta, which is based on financial statements and macro-market data.^{21) 22)}

As Botherson et. al. (2013)²³⁾ stated, the forward-looking betas are unobservable, so practitioners are forced to rely on proxies of various kinds. Often times, beta are derived from historical data. The usual methodology is to estimate beta as the correlation coefficient of market returns. As such, we collected the return of construction sponsors during a set of time period (t) and regressed them on market return in a pooled manner:

$$R_{(i \times t)t} = \alpha_a + \beta_a \cdot R(m)_t \quad (4.3)$$

20) When calculating pooled average beta of construction shareholders, redundant companies are treated as one.

21) Toms, S., 2012, 'Accounting based risk measurement: An alternative to CAPM derived discount factors'

22) Brearley, R. Myers, S. and Allen, F, 2006, 'Principles of Corporate Finance'

23) W. Todd Botherson, Kenneth M. Eades, Robert S. Harris, and Robert C. Higgins, 2013 'Best Practices in Estimating the Cost of Capital: An Update., Journal of Applied Finance

$R_{(ixt)t}$ = set of returns on construction sponsor's stock 'i' in time period 't'

R_{mt} = return on market portfolio in period 't'

α_a = correlation coefficient of constant

β_a = average beta of construction sponsors who joined BTL

From the total 426 contracted BTL projects, 458 construction equity holders are extracted²⁴⁾. Based on financial statements of 458 construction firms, the time-series return on equity (R_{it}) from 2000 to 2015 (15 years) are calculated. The 31 firms that showed exceptionally high or low average returns are deleted as outlier. (Please refer to the appendix 1). In the above formula, 'i' represents 427 construction sponsors and 't' represents 15 years of time period. Brotherson(2013) maintained that "increasing the number of time period used in the beta estimation may improve the statistical reliability of the estimate but risks including stale, irrelevant information. Similarly, shortening the observation period from monthly to weekly, or even daily, increases the size of the sample but may yield observations that are not normally distributed and may introduce unwanted random noise." Most of the construction sponsors being unlisted companies, the financial statements are announced yearly basis. And the observation time slot is 15 years, generating 15 ROEs for each company. By pooling the 427 construction sponsors each of which has 15 years of ROEs²⁵⁾, we can get total 6,042 number of ROEs. The pooled ROEs are expressed as a set of returns on stocks 'i' in time period 't' in above formula ($R_{(ixt)t}$). The average of pooled 6,042 number of estimated ROEs is 10.19%.

Theories say R_m is the return on 'market portfolio', which is not observable portfolio and consisted of all risky assets in proportion to their importance in world wealth. In practice, a variety of stock market index are used as a proxy for market portfolio. However, as the above beta are estimated based on the accounting data in this paper, we estimate R_m in a consistent way, using financial data: The weighted

24) Source: internal database of PIMAC and Ministry of Strategy and Finance

25) Some companies do not have 15 years of business operations, so the number of ROEs that used is 6,042 instead of 6,405

average returns on equity (ROE) of over 21,200 companies that reported financial statements are calculated from 2000 to 2014 (See appendix 4). The 15 years' average of estimated market return (R_{mt}) is 7.87%, which is 2.32% lower than that of construction sponsors. The construction industry is known to have higher risk than market average.

The pooled set of returns ($R_{(ixt)t}$) are regressed on market returns (R_{mt}) to get the average beta of construction sponsors that joined BTL. Additionally, three dummy variables that stratify projects into four sectors are included to see how much systematic differences exist among facility sectors. Environmental facility sector being the base case, the 'mil' represents for military, 'edu' for education, and 'cult' for culture facilities.

Table 3. Estimated average beta of construction sponsors

VARIABLES	(1) $R_{(ixt)t}$	(2) $R_{(ixt)t}$
R_{mt}	1.144*** (0.156)	1.143*** (0.156)
mil		0.0190 (0.0282)
edu		-0.0156 (0.0134)
cult		0.00438 (0.0138)
Constant	0.0116 (0.0158)	0.0142 (0.0170)
Observations	6,042	6,042
R-squared	0.005	0.006

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

We got a pretty robust correlation coefficient for R_{mt} , which value is about 1.143 and reliable under 99% of confidence level. It means the average beta of construction sponsors that joined BTL is estimated as 1.143. All of the dummy variables didn't

show any significant coefficient. So, the return of construction sponsors in different facility sectors didn't have significant difference against the change of market return, assuming other conditions being equal. This result means there doesn't exist the level of difference in constant, not the difference of slope (estimated beta).

However, what we are more interested is the difference of beta among facility sectors. It is possible that the systematic risk of construction sponsors may be different among facility types. To check this possibility, the similar process is performed again by separate facility sector basis. Instead of pooling all BTL construction sponsors into one portfolio, we classified the construction sponsors into four categories and regressed the pooled return of each sector's construction sponsors on market return four times, in separate.

$$R(g)_{(i \times t)t} = \alpha_g + \beta_g \cdot R(m)_t \quad (4.4)$$

$R(g)_{(ixt)t}$ = set of returns on construction sponsor's stock 'i' within facility sector 'g' at time period 't'

R_{mt} = return on market portfolio in period 't'

α_g = correlation coefficient of constant in facility sector 'g'

β_g = average beta of construction sponsors within facility sector 'g' (i.e. β_{env} means the average beta of construction sponsors that joined environmental BTL projects, β_{mil} means that of military, β_{edu} means that of schools, and β_{cult} means that of culture BTL projects)

The regression result is summarized in the below table:

Table 4. Estimated average beta of construction sponsors in each of facility sectors

VARIABLES	(1) $R(env)_{(ixt)t}$	(2) $R(mil)_{(ixt)t}$	(3) $R(edu)_{(ixt)t}$	(4) $R(cult)_{(ixt)t}$
R_{mt}	1.158*** (0.285)	1.288*** (0.474)	1.177*** (0.259)	1.102*** (0.247)
Constant	0.013 (0.0263)	0.0218 (0.0544)	-0.0078 (0.0259)	0.0218 (0.025)

Observations	1,600	1,041	2,521	895
R-squared	0.007	0.002	0.008	0.02

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

$R(env)_{(ixt)t}$ represents the set of returns on construction sponsors that joined environmental BTL projects from 2000 to 2014. The correlation coefficient of R_{mt} on $R(env)_{(ixt)t}$ in model (1) means the estimated average beta of construction sponsors that joined environment sector BTL projects. The average beta of constructors of environmental facility sector (β_{env}) is estimated as 1.158. The same logic applies to other sectors, resulting in the average construction shareholder's beta in military sector (β_{mil}) is estimated as 1.288, that for education sector (β_{edu}) is 1.177, and that for culture sector (β_{cult}) is 1.102. As is mentioned, the motivation for the stratified pooling approach is to see whether there exist a different level of systematic risks among facility sectors. The equality of betas are tested using 2-tailed z-test (see appendix 2). It is concluded that there is no evidence that betas of all facility sectors (β_g) are equal to pooled average (β_a) under 99% of confidence level, except that of environment. However, the beta of environment is different from the pooled average (β_a) under 95% of confidence level. So, it is determined that stratified betas (table 4) for respective facility sectors (β_g) should be used to estimate each of the costs of equity capitals.

The betas are measured from construction sponsors because they are the most comparable companies that have isomorphic systematic risk with BTL projects during construction period. But the above estimated average betas of the four facility sectors (β_{env} , β_{mil} , β_{edu} , β_{cult}) reflect the financial ratio of construction companies, not that of BTL project companies. So, the beta should be adjusted reflecting the average financial ratio of BTL projects because our goal is to get the optimal rate of return for BTL investment during construction period. The HAMADA model²⁶⁾ can be

26) Hamada, R.S. (1972) "The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stocks," The Journal of Finance

applied to revise estimated sector betas (β_g), reflecting the leverage ratio of BTL projects different from that of construction sponsors. First, un-levered betas of four facility sectors are calculated as follows:

$$Unlevered \beta_g = \frac{\beta_g}{[1 + (1 - \tau) \times \frac{\sum_{i=1}^n D_i}{\sum_{i=1}^n E_i}]} \quad (4.5)$$

D_i = average (from 2012 to 2014) debt amount of construction sponsors 'i'

E_i = average (from 2012 to 2014) equity amount of construction sponsors 'i'

τ = corporate tax rate

The sum of debt and equity amounts are calculated based on the same sample of 458 construction sponsors. The weighted average leverage ratio of 458 construction

sponsors ($\frac{\sum_{i=1}^n D_i}{\sum_{i=1}^n E_i}$) is 2.092X. On the basis of this ratio, the un-levered betas of

facility sectors are calculated. And again, the weighted average leverage ratio of 426 BTL projects is applied to get revised average betas in each of sectors (β_g') as below:

$$\beta_g' = Unlevered \beta_g \times [1 + (1 - \tau) \times \frac{\sum_{j=1}^m D_j}{\sum_{j=1}^m E_j}]$$

D_j = debt amount of BTL project 'j'

E_j = equity amount of BTL project 'j'

τ = corporate tax rate

The leverage ratio of respective facility sectors are measured based on historical contract data. The ratio of environment is 10.43X, that for military is 15.97, that for education is 10.82, and that for culture facility is 12.42. Along with the leverage

ratio, 24.2% of marginal corporate tax rate of Korea is applied to the formula, resulting in below table:

Table 5. Beta revision reflecting difference in financial ratio

Facility sectors	Beta (β_g) (constructor's leverage ratio)	unlevered Beta	Revised Beta (β_g') ²⁷⁾ (BTL leverage ratio)
Environment	1.158	0.4478	3.9876
Military	1.288	0.4981	6.5272
Education	1.177	0.4551	4.1885
Culture	1.102	0.4261	4.4389

A risk free rate (R_f) is also needed to calculate the cost of equity capital (k_e). Brotherson et. al. (2013) states that "long-term bond yields more closely reflect the default-free holding period returns available on long-lived investments and thus more closely mirror the types of investments made by companies. The survey results reveal a strong preference on the part of practitioners for long-term bond yields. Most of the corporations and financial advisors use T-bond yields for maturities of ten years or greater, with the ten years rate being the most popular choice. Many corporations said they matched the term of the risk-free rate to the tenor of the investment. In contrast, a third of sample books suggested subtracting a term premium from long-term rates to approximate a shorter term yield. Half of books recommended long-term rates but were not precise in the choice of maturity." Vecchi V, et. al. (2013) used the UK government bonds maturing in 15, 20, 25, and 30 years to estimate sponsor's cost of equity for health PPP. It may be said that there is not a decisive theory for selecting a risk free rate (R_f).

27) The revised levered beta of BTL in construction period seems irrationally high. It is due to the nature of BTL business, which allows high leverage ratio because the SPC is destined to be liquidated when concession is over. Especially, BTL investment is recognized safe and thus, around 5% of minimum equity capital is allowed and remaining 95% is funded through debt. The revised levered beta is high due to the high leverage ratio of BTL however, when the estimated cost of equity is applied to WACC, cost of equity takes small portion, and thus the final result of calculated WACC makes rational range, which is not much different from that of construction industry. Assuming WACC is calculated without reflecting high leverage ratio of BTL, the WACC of environment is 5.72%, military is 6.15%, education is 5.74%, and culture is 5.97%.

In general, BTL contracts determines contract IRR by adding risk spread (α) to base rate, which is five years sovereign bond. The risk spread (α) doesn't change during the whole investment period, while the base rate (five years bond rate) resets every five years. The intention seems that the reset duration is aligned with base rate period. Considering this term structure, five years sovereign bond rate is selected as a proxy for risk free rate in this research. In line with this observation, all of five-year sovereign bond rates at respective contract dates are collected and averaged by sectors to get the average risk free rates (R_f) in four sectors. The result of average risk free rate in environment is 4.56%, that in military is 4.76%, that in education is 4.95%, and that in culture sector is 4.70%.

The market risk premium is a spread between expected return of market portfolio and risk free rate, meaning a compensation for taking a unit of systematic risk.

$$\text{Market Risk Premium}(MP) = [E(R_m) - R_f] \quad (4.6)$$

In theory, the market premium should be measured from 'ex-ante' market return. However, it is technically hard to identify 'market portfolio' and even impossible to directly observe 'expected' market return. For this reason, long-term ex-post average of stock market return minus long-term average risk free is used as a proxy to market risk premium. A possible difference between 'expected' and 'realized' market return can be adjusted by using a long-term time-series data and thus, deleting the systematic bias of investors. For example, Mehra and Prescott(1985) analyzed 100 years of past time-series data of United States, and Siegel(1998) worked with 50 years of data of United Kingdom after World War II. Usually Ibbotson or Morningstar data are used as a reference for market premium. Several studies suggested the market premium of Korea from 5.4 to 8.1% range. Choi(2008) suggested 6%, Kang(2008) maintained 5.45~6.38%, Park(2009) stated 5.67~8.06%, and Damodaran(2012) 6.11%. Among the range of research results, 6% of market risk premium is selected in this paper.

The revised average beta of four sectors (β_g'), risk free rate (R_f), and the market risk premium are substituted into CAPM to estimate the optimal level of returns on equity (k_e) for BTL projects during construction period in the four sectors. The input and output data are summarized as follows:

Table 6. Summary of input and output data for CAPM

Items	Input data		Notes
Revised average sector beta (β_g')	environment	3.9876	average beta for each of facility sectors, measured from construction sponsors, and additionally revised reflecting the leverage ratio of BTL projects
	military	6.5272	
	education	4.1885	
	culture	4.4389	
Risk free rate (R_f)	environment	4.563%	average of five years sovereign bond rate, measured at all of individual contract dates and averaged by sectors
	military	4.760%	
	education	4.946%	
	culture	4.697%	
Market premium [$E(R_m)-R_f$]	6%		Selected from researches
Estimated return on equity capital for BTL sectors during construction period $E_t[k_e]$	environment	28.4886%	$E_t[k_e] = R_f + [E(R_m) - R_f] \times \beta_g'$
	military	43.9235%	
	education	30.0769%	
	culture	31.3304%	

In general, several tranches of loans are combined for the financing of infrastructures to allocate different risk-return profiles among lenders with diverse risk appetites. Typical example of funding structure includes senior and subordinated loans with combination of fixed and floating interest rates. However, as is discussed, the BTL investment is recognized as buying a sovereign bond that has additional risk of construction and operation. For this reason, most of risk averse investors such as pension funds or insurance companies are eager to invest in BTL and thus, a single tranche of senior loan can fund the whole amount of debt. That is why the loan

structure of BTL is very simple: a single tranche of senior loan. The Standard BTL Concession Agreement of Korea defines the cost of debt as three years AA⁻ bond rate plus spread. The three years AA⁻ bond rate is determined by the five days average of market data ahead of contract date, and revised in every five years reflecting market rate. The average cost of debt (k_d), in each of four sectors, are measured based on the 397 BTL contract data²⁸⁾ from 2005 to 2014. The result of average cost of debt in environment sector is 6.828%, that in military is 7.321%, that in education is 6.833%, and that in culture is 7.236%.

The equity ratio (K) is the relative proportion of equity used to finance total asset. The amounts of equity and debt are often taken from the firm's balance sheet or market values for both, if a company's equities are publicly traded. In the BTL case, few companies are listed and thus, book values at contract points are used in calculating equity ratios. The equity ratio (K) measures the proportion of the total assets that are financed by shareholders, as opposed to creditors, indicating the level of leverage used by a company. Generally, infrastructure funding requires over 20% of equity ratio to secure debt against business distress. However, thanks to the stable business nature of BTL, minimum 5% of equity ratio is allowed by PPP Basic Plan. In practice, market lenders require 5% ~10% of equity ratio for BTL in Korea. The two major sorts of equity investors are construction sponsors and financial sponsors: the former are interested in Engineering, Procurement, and Construction (EPC) contracts whereas, the latter are interested in long-term financial return after construction. For this reason, during the construction phase, construction sponsors lead a project, holding major share of equity. When construction risk is gone and operation begins, financial sponsors usually purchase the major share of equity from construction sponsors. The average equity shares (K) of four sectors are measured based on the 397 BTL contract data from 2005 to 2014. The result of average equity share (K) in environment sector is 8.75%, military is 5.89%, education is 8.46%, and culture is 7.45%.

28) 397 projects that provided cost of debt information among 426 of population

The standard concession duration of BTL in Korea is 20 years, which is far longer than five years and thus, it is appropriate to additionally consider a long-term investment premium. The 20 years sovereign bond was first issued in 2006 in Korea. The average term spread between five and twenty years bonds from 2006 to 2014 is observed as 0.4570%. (refer to the appendix 3)

Finally, the optimal average returns during construction period ($r^*_{c,t}$) in four facility sectors are estimated, applying above assumptions. The optimal level of returns in construction phase are estimated from 7.67% to 8.27% as follows:

Table 7. The optimal return during the construction period ($r^*_{c,t}$)

Sectors	$E_t[k_e]$	$E_t[k_d]$	K	WACC	term premium	$r^*_{c,t}$
environment	28.4886%	6.828%	8.75%	7.22%	0.4570%	7.67%
military	43.9235%	7.321%	5.89%	7.81%		8.27%
education	30.0769%	6.833%	8.46%	7.29%		7.74%
culture	31.3304%	7.236%	7.45%	7.41%		7.87%

2. The optimal return in operation period ($r_{o,t}^*$)

(1) Merton's option theory²⁹⁾ and BTL risk premium in operation period

The Merton Model was developed in early 1970s and has been applied to the evaluation of credit risk in corporate's debt. A credit risk is the uncertainty, surrounding an entity's ability to service its debt and obligations. Prior to default, there is no way to determine unambiguously an entity whether it will default or not. The best ex-ante efforts that we can do is only to make a statistic assessment of the default probability based on other benchmark projects. As a matter of course, a spread over the default-free rate of interest should be paid to compensate the uncertainty proportionally to its default probability. In similar context, Merton's option theory, which was originally designed to measure credit risk spread of corporate's debt, may be able to provide a practical tool for measuring operation risk of BTL contract because the operation cash flow of BTL is isomorphic to risky corporate bond.

The Merton model assumes there be a single class of bond with par-value of 'L' and maturity of 'T'. If the value of asset falls behind the debt amount, company will go default. The model is based on the assumption that market is frictionless and the movement of risky asset follows geometric brown motion:

$$dA_t = \mu A_t dt + \delta A_t dW_t, \quad A_0 > 0$$

Where, the μ is the mean rate of return on the assets and δ is the asset volatility

Basically, the model considers a company's equity as a call option on its asset. While the debt holder's pay-off at the exercising date has isomorphic position to owning a safe debt and shorting a put option underlying the corporate's asset³⁰⁾.

29) Robert C. Merton (1972), On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, Journal of Finance

30) Wang, Y., 2009, Structural Credit Risk Modeling, Society of Actuaries.

$$D_T = \min(L, V_T) = L + \min(V_T - L, 0) = L - \max(L - V_T, 0) \quad (4.7)$$

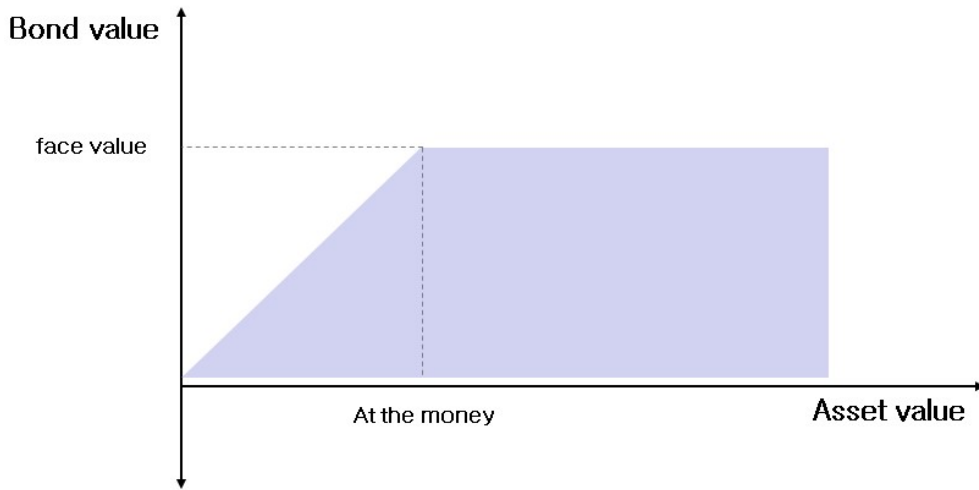
D_T = the payoff of debt at maturity 'T'

V_T = the value of underlying asset

L = the face value of debt

In summary, a corporate debt has a payoff position of shorting a put option and possessing a (risk free) bond.

Figure 7. Bond holder's pay-off position at maturity



(Source: Anthony Saunders and Linda Allen (2012), Credit Risk Measurement)

The present value of debt at forecasting point 't' (D_t) can be priced by applying the risk neutral expected discounted payoffs of debt and the Black-Scholes option model as follows:

$$D_t = PV[L - \max(L - V_t, 0)] \quad (4.8)$$

$$= Le^{-r(T-t)} - P_t(V_t, L, T-t, r)$$

$$= Le^{-r(T-t)} - [Le^{-r(T-t)} N(-d_2) - V_t N(-d_1)]$$

$$= Le^{-r(T-t)} [1 - N(-d_2)] + V_t N(-d_1)$$

$$= Le^{-r(T-t)} N(d_2) + V_t N(-d_1)$$

$$d_1 = \frac{\ln(V_t/L) + (r + \delta^2/2)(T-t)}{\delta \sqrt{T-t}} \quad d_2 = \frac{\ln(V_t/L) + (r - \delta^2/2)(T-t)}{\delta \sqrt{T-t}}$$

D_t = the present value of debt at time 't'

V_t = the spot value of underlying asset at 't'

L = the face value of debt

$N(d)$ = the cumulative probability distribution for a variable that has a standard normal distribution with zero mean and one standard deviation (Please see appendix 10)

At the same time, the present value of debt (D_t) can be expressed based on the yield to maturity (y) and the face value (L) of debt ($D_t = L e^{-y(T-t)}$). This formula can be transformed from the viewpoint of 'y' variable ($y = \frac{-\ln(D_t/L)}{(T-t)}$). By substituting $[L e^{-r(T-t)} N(d_2) + V_t N(-d_1)]$ to the current value of debt (D_t), the yield to maturity (y) can be expressed as below:

$$y = -\frac{1}{T-t} \times \ln\left[\frac{L e^{-r(T-t)} N(d_2) + V_t N(-d_1)}{L}\right] \quad (4.9)$$

The credit risk spread (s) is a difference between yield to maturity and risk free rate ($s = y - r$). By substituting above drawn formula to yield to maturity, the credit spread can be expressed as below:

$$s = y - r \quad (4.10)$$

$$= -\frac{1}{T-t} \times \ln\left[\frac{L e^{-r(T-t)} N(d_2) + V_t N(-d_1)}{L}\right] - r$$

$$= -\frac{1}{T-t} \times \ln\left[N(d_2) + \frac{V_t}{L e^{-r(T-t)}} N(-d_1)\right]$$

If, $\frac{V_t}{L e^{-r(T-t)}}$ is substituted by $(\frac{1}{\lambda})$, then

$$= -\frac{1}{T-t} \times \ln\left[N(d_2) + \frac{1}{\lambda} N(-d_1)\right]$$

This formula represents the risk spread of corporate's bond. To make it a simpler form, if we put $t = 0$, the formula can be simplified as follows³¹⁾:

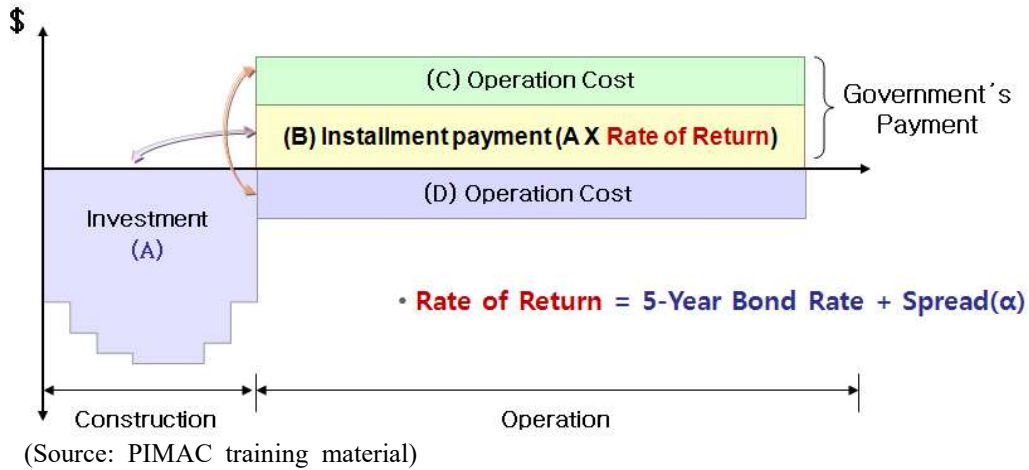
$$s = -\frac{1}{T} \times \text{Ln}[N(d_2) + \frac{1}{\lambda} N(-d_1)] \quad (4.11)$$

where, ' λ ' is the leverage ratio

The risk spread(s) is a function of loan and asset value ratio, risk free rate, maturity, and degree of asset volatility.

As is discussed, the business nature of BTL during the operation period is similar to holding a corporate bond that has an uncertainty mainly caused by operation cost.

Figure 8. SPC's cash flow in BTL investment



In general, the revenue of BTL is pre-determined government's payment that is comprised of 'installment payment(B)' plus 'operation cost(C)'. The 'installment payment(B)' is an equal annual payment that includes principal and interest for the compensation of the 'initial capital investment(A)' plus project return. The sum of government's payment(B+C) is pre-determined by contract because the 'operation cost (C)' is a budget basis, which does not change even though the 'real operation cost(D)'

31) This formula is drawn from Black-Scholes put option pricing model, and thus the expression is different from the original Merton model. However, the original Merton formula is used for the calculation in this research. Please refer to (appendix 10)

changes. As a result, ax-ante expectation of net operation cash flow of BTL project is the 'installment payment(B)'³²⁾.

It is highly probable that the ex-post net operating cash flow deviates from the 'installment payment(B)' because the 'real operation cost(D)' will probably deviate from the 'budget operation cost(C)'. Procuring authority pays operation cost up to the pre-determined budget. And thus, if the 'real operation cost(D)' is bigger than the 'budget(C)', it results in cost-overflow, and at the same time, nibbling the installment payment (B). If the 'real operation cost (D)' is smaller than the 'Budget operation cost (C)', procuring authority does not pay for the saved cost.³³⁾ As a result, the payoff of BTL project during operation period is similar to investing in corporate loan that has a degree of risk due to the operation cost overrun.

There are additional factors that may cause additional volatility such as irregular capital investment or government's payment deduction. One of the examples is the uncertainty in government's payments even though the possibility is not substantial. As the pre-determined revenue is paid by government, regardless of usage level in BTL, private investors may have moral hazard that they do not operate the subject facility not in good condition as is defined by the concession agreement. As a result, most of BTL contracts include such clause that a procuring authority can deduct the payment based on the performance assessment when the performance of target facilities do not meet the pre-determined criteria. This clause is indispensable from the public points of view to control above mentioned moral hazard whereas it may cause a volatility in net operation income of SPC.

In the BTL context, present value of debt at forecasting (t=0) point is same as ex-ante value of asset, which means a BTL project company provides the same position as investing to a company comprised of same amount of asset and debt. In

32) Net operating cash flow = (B)Installment payment + (C)Budget operating cost - (D)Actual operating cost. Assuming the ex-ante expectation of (C)Budgetary operating cost equals to (D)Actual operating cost, E(Net operating cash flow) becomes same as the (B)Installment payment

33) In practice, if smaller amount of actual operation cost(cost saving) is expected, SPC makes pre-expenses or promotes early execution

summary, BTL project's net asset value during the operation period is same as holding series of bonds like installment payments, which value depends on the deviation of operation cost plus additional volatility.

$$\text{Value of BTL project} = \text{PV} [(\text{B})\text{Installment payment} + (\text{C})\text{Budget operation cost} - (\text{D})\text{Actual operation cost}] \pm \text{other volatility}$$

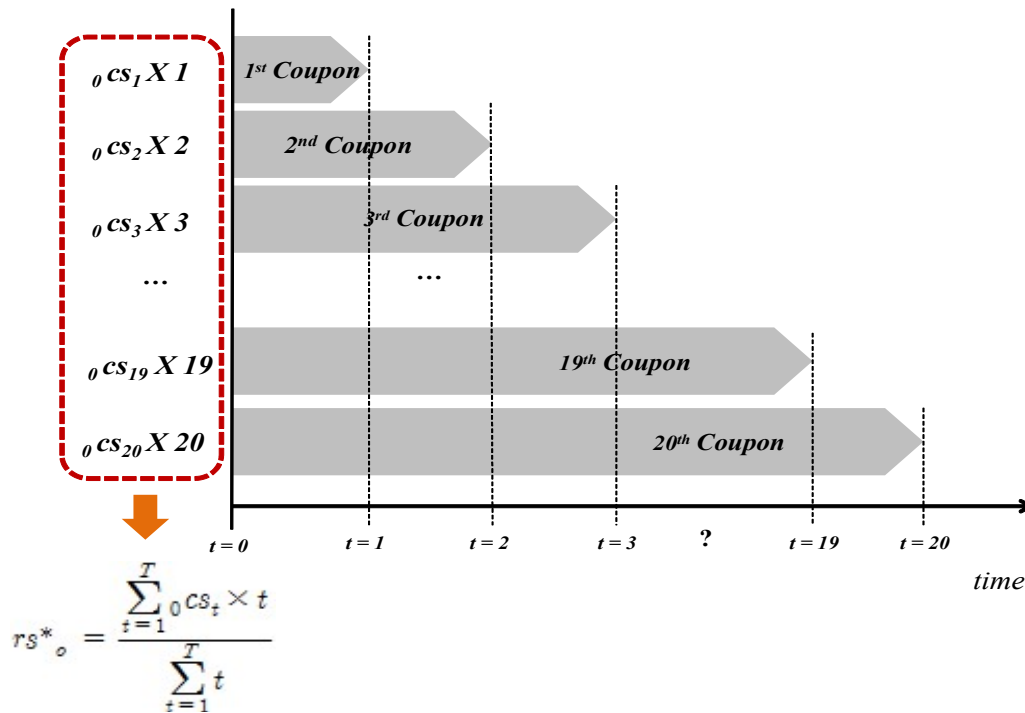
$$\text{Degree of asset value volatility} = f [(\text{C})\text{Budget operation cost} - (\text{D})\text{Actual operation cost} \pm \text{other volatility}]$$

It being said, the Merton's option model is one of the candidates applicable to the risk spread(s) measurement of BTL project during operation period. Although, there being several sorts of credit risk theories, the Merton's model is simple in concept, and requires only three main variables: degree of asset volatility, a value ratio between asset and debt, and investment maturity. When a risk spread is estimated, the optimal BTL return during operation period can be simply calculated by adding risk free rate (R_f) and long term investment premium.

(2) Estimation of the optimal return ($r^*_{o,t}$) in operation period

Contemplating on the BTL payment structure in detail, the government's installment payment is comprised of 'T' numbers of coupons³⁴⁾, each of which have different maturities. The risk spread in each of coupons should be calculated in separate because they have different maturities. The measuring approach of BTL risk spread during operation period is illustrated as follows:

Figure 9. Estimation of BTL risk spread during operation period ($r^*_{o,t}$)



${}_0cs_t$: coupon risk spread with 't' year's maturity (measured using Merton's option model)

rs^*_o : BTL risk spread in operation period

T : concession period (generally, 20 standard years in Korea)

The individual coupon spread (${}_0cs_t$) is estimated, applying Merton's option model as follows³⁵⁾:

34) 'T' represents operation maturity in years determined by BTL contract

35) The original form of Merton theory is used in calculating the coupon risk spread (please refer to the appendix 10)

$${}_0cs_t = -\frac{1}{t} \times \text{Ln}[N(d_2) + \frac{1}{\lambda}N(-d_1)] \quad (4.12)$$

where, ' λ ' is the leverage ratio

And the optimal risk spread (rs^*_o) during the whole operation period is calculated by weighted average of coupon spreads (${}_0cs_t$) based on the maturity of individual coupons. (refer to the appendix 6)

$$rs^*_o = \frac{\sum_{t=1}^T {}_0cs_t \times t}{\sum_{t=1}^T t} \quad (4.13)$$

The estimation of optimal BTL return in operation period ($r^*_{o,t}$) begins from measuring coupon risk spreads in operation period (${}_0cs_t$). As is discussed, a BTL investor's position is similar to buying a company at a price of bond and shorting a put option on a company's asset. If the company's asset value goes below debt amount, the debt holder has to bring the ownership of the company. It means the exercise price at maturity is the par value of debt, and in case a company's value falls short of debt payment, it goes default. The payoff of the debt holder, in this case, will be the asset value minus face value of debt. Applying the real option theory to BTL case, it can be rationally assumed that the present asset value equals to that of bond³⁶⁾ at forecasting point because the expected operation cost is equal to actual in ax-ante basis³⁷⁾. It can be translated that BTL investment has 100% of loan to value ratio (LTV) at forecasting phase. Because the necessary input value to Merton model is the loan to value ratio (LTV), not an absolute value, simply 'one' can be applied. And, it is considered as a default when the value of asset goes under loan balance, meaning the loan to value (LTV) becomes higher than 100%.

36) The net value becomes the sum of present value of installment payment in BTL contract, which is also 'at the money' status in option theory

37) In fact, there exists no actual cost at forecasting point

The remaining necessary variable is the degree of asset volatility. In general, the change of an asset value depends on the volatility of future cash flow (net operation income in a narrow term), given a constant discount rate. Let's identify $E(V_j)$ as an expected asset value of a BTL project 'j', which value is dependent on expected net operation income $E(NOI_{jt})$ and discount rate that is assumed as a given risk free rate. It turns out the $E(V_j)$ can be expressed as a function of $E(NOI_j)$.

$$E(V_j) = f[E(NOI_{jt}), \bar{r}] = f[E(NOI_{jt})] \quad (4.14)$$

The change of asset value at time 't' (ΔV_{jt}) can be derived as a function of deviation on net operating income (ΔNOI_{jt}) as below:

$$E(V_{jt}) = E(V_{j(t-1)}) + E(\Delta NOI_{jt}) \quad (4.15)$$

$$E(V_{jt} - V_{j(t-1)}) = E(\Delta V_{jt}) = E(\Delta NOI_{jt})$$

Again, the NOI can be decomposed into revenue and operation cost. The pre-determined revenue payment is the government's responsibility in BTL contract and the government's credit risk is deemed as risk free, which means $E(\Delta Revenue_{jt})$ is considered as almost zero. So, the change of asset value (ΔV_j) mainly depends on the change of operation cost.

$$E(NOI_{jt}) = E(Revenue_{jt} - Operating\ cost_{jt})$$

$$E(\Delta NOI_{jt}) = E[\Delta(Revenue_{jt} - Operating\ cost_{jt})]$$

$$\text{Assuming, } \Delta Revenue \approx 0,$$

$$E(\Delta NOI_{jt}) \approx -E(\Delta Operating\ cost_{jt})$$

$$E(\Delta NOI_{jt}) \approx |E(\Delta Operating\ cost_{jt})|$$

$$\therefore E(\Delta V_{jt}) = E(\Delta NOI_{jt}) \approx |E(\Delta Operating\ cost_{jt})|$$

Now, what we need to know is the standard deviation of asset value in each of

facility sectors ($\delta(V_g)$). The straight-forward way is to measure time-series deviation of NOI_{jt} of each project ($\delta(NOI_{jt})$). However, a critical problem arises: as the BTL was introduced to Korea in 2005, the earliest generation projects commenced their operation in 2006. And so, the longest operation duration until 2014 is only nine years, which is not enough time series for statistically reliable analysis.

The alternative approach for measuring the degree of unexpected volatility of NOI may be a cross-sectional analysis. The basic assumption is that a rational investor will forecast NOI of a project from the track-record of comparable projects in operation. The common knowledge of business states that operation cost is divided into fixed and variable that is proportional to revenue. And companies within similar business have typical cost structure that can be formulated based on the financial data of similar business sector. On the basis of these assumptions, the NOI estimation formula is obtained through a cross-sectional regression, and again the estimated NOI is obtained by substituting actual revenue into the NOI estimation line. The actual NOI data is directly collected from financial statements. The unexpected deviation of NOI is measured by calculating the difference between estimated and actual NOI.

$$NOI_j = a_g + b_g \cdot S_j + r_j \quad (4.16)$$

$$E(NOI_j) = \hat{a}_g + \hat{b}_g \cdot S_j$$

NOI_j : net operation income of project 'j'
 S_j : revenue amount of project 'j'
 r_j : residual value of estimated NOI in project 'j'

Operation risk spread is calculated by consistent categories to construction risk measurement: environment, military, school, and culture. As the operation cost structures are different depending on facility sectors, the formula of NOI will probably be different among sectors. For example, library may have bigger volatility than simple sewage pipeline systems. For this reason, not only NOI estimation lines, but also the standard deviations of asset values are measured in separate sectors basis. The 49 SPCs from environment, 47 from military, 139 from education, and 26 from

culture facility sectors are selected as samples for regression analysis.³⁸⁾ The net operation incomes (NOI) are regressed on the revenues of 2014³⁹⁾ stratified by four facility sectors, to get four separate regression lines that explain the relation between revenues and net operation incomes. The 'NOI(g)_j' represents the net operating income of project 'j', and the 'S_j' represents the revenue amount of project 'j' in fiscal year of 2014.

Table 8. Regression analysis for NOI estimation

VARIABLES	(1) NOI(env) _j	(2) NOI(mil) _j	(3) NOI(edu) _j	(4) NOI(cult) _j
S _j	0.781*** (0.0599)	0.809*** (0.0261)	0.678*** (0.063)	0.722*** (0.0764)
Constant	-4.59E+07 (1.92E+08)	-2.509e+08** (1.08E+08)	-4.55E+07 (1.93E+07)	-2.006e+08* (1.17E+08)
Observations	49	47	139	26
R-squared	0.895	0.937	0.807	0.827

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

The estimated correlation coefficients for revenues (\hat{b}_g) resulted in a range of 0.678 ~ 0.809, and those of constants (\hat{a}_g) are all negative values. These directions are rational because the expected NOI should be positively proportional to sales while it should be negative when sales is zero due to a fixed cost. As the regression models are simple (one-factored) ones, we rely on the R-squares, all of which are over 0.8. The regression lines are illustrated as follows:

38) Samples are selected when projects meet both criteria of in operation status and showing positive NOI. Which means prospective investors will rationally expect that NOI would be positively correlated with revenue. Please refer to the (appendix 5) for detail statistics.

39) The financial data of 2014 is selected for cross-sectional regression because the latest data provide the biggest number of samples in operation status, and the project companies that had begun operation before 2014 will have a stable NOI. A SPC in early operation status usually shows abnormally low or negative profit due to some extraordinary expenses in beginning stage, such as expenses for testing period or capital expenses etc. In Korea, 18 BTL projects commenced their operation in 2014, and 43 projects begun in 2013, all of which may have been dropped if 2014 data is not selected.

Figure 10-1. NOI estimation line (env)

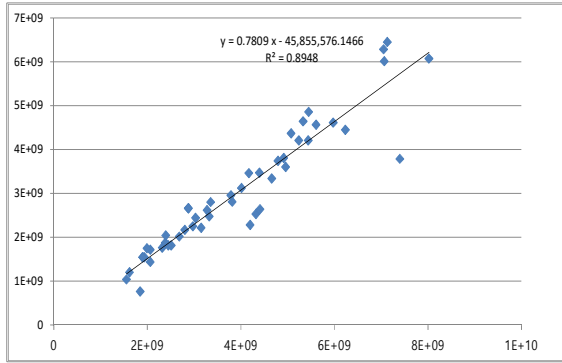


Figure 10-2. NOI estimation line (mil)

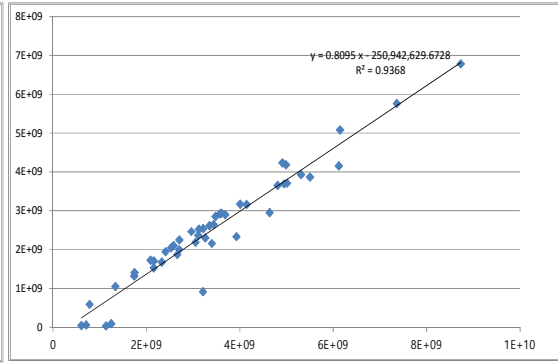


Figure 10-3. NOI estimation line (edu)

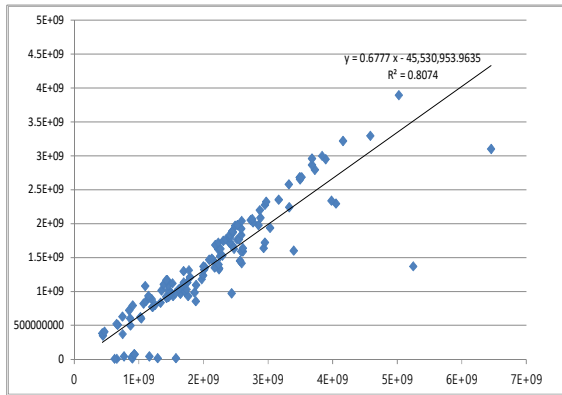
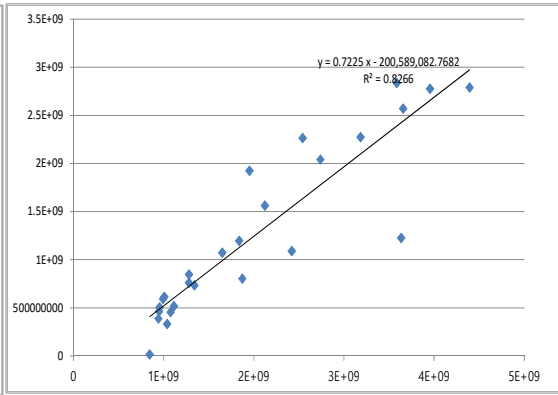


Figure 10-4. NOI estimation line (cult)



The advantage of measuring volatility from NOI instead of operation cost is that it can capture deviations generated from operation cost, as well as other unexpected factors such as capital expense, revenue deduction for bad performance and so on.

Again, the actual revenues of 2014 (S_j) are substituted into the regression lines to get expected operation incomes of four sectors. The expected operation incomes are subtracted from the actual to get the residual values (\tilde{r}_i), which is considered as the unexpected deviation of net operation income $E(\Delta NOI_j)$ and same with the unexpected deviation of asset value $E(\Delta Asset\ value_j)$ under BTL concession.

$$NOI_j - E(NOI_j) = \tilde{r}_j \quad (4.17)$$

NOI_j : actual net operating income (reported by F/S) of project 'j'

$E(NOI_j)$: estimated net operating income of project 'j' based on regression line

\tilde{r}_j : estimation error on NOI_j (unexpected deviation of NOI_j)

Table 9. Summary of the correlation coefficients, used for NOI estimation

Correlation coefficients	environment	military	education	culture
\hat{b}_g	0.781	0.809	0.678	0.722
\hat{a}_g	-4.59e+07	-2.509e+08	-4.55e+07	-2.006e+08

Now we can get the unexpected deviation of asset value $E(\Delta Asset\ value_j)$. However, it should be transformed into a ratio to asset amount by dividing with asset value.

$$\% \Delta of Asset\ Value_j = \frac{\Delta of Asset\ Value_j}{Asset\ value_i} = \frac{\tilde{r}_i}{Asset\ value_i} \quad (4.18)$$

There are two reasons for the re-scaling. First of all, the unexpected deviations of asset values ($\Delta Asset\ value_j$) are estimated based on the projects with different investment scales, so the observed unexpected asset value volatilities in amount basis can't be directly compared among projects. Second, the unit of standard deviation (δ) identified in Merton's model is the standard deviation in asset value. So, the unexpected deviation of asset value ($\Delta Asset\ value_j$) should be re-scaled to be expressed by a distribution and standard deviation in asset value unit.

$$\delta(\% \Delta of Asset\ Value_j) = \delta \left[\frac{\Delta of Asset\ Value_j}{Asset\ value_i} \right] = \delta \left(\frac{\tilde{r}_i}{Asset\ value_i} \right)$$

The result of unexpected deviation of asset value in percentage $\left(\frac{\Delta of Asset\ Value_j}{Asset\ value_i} \right)$

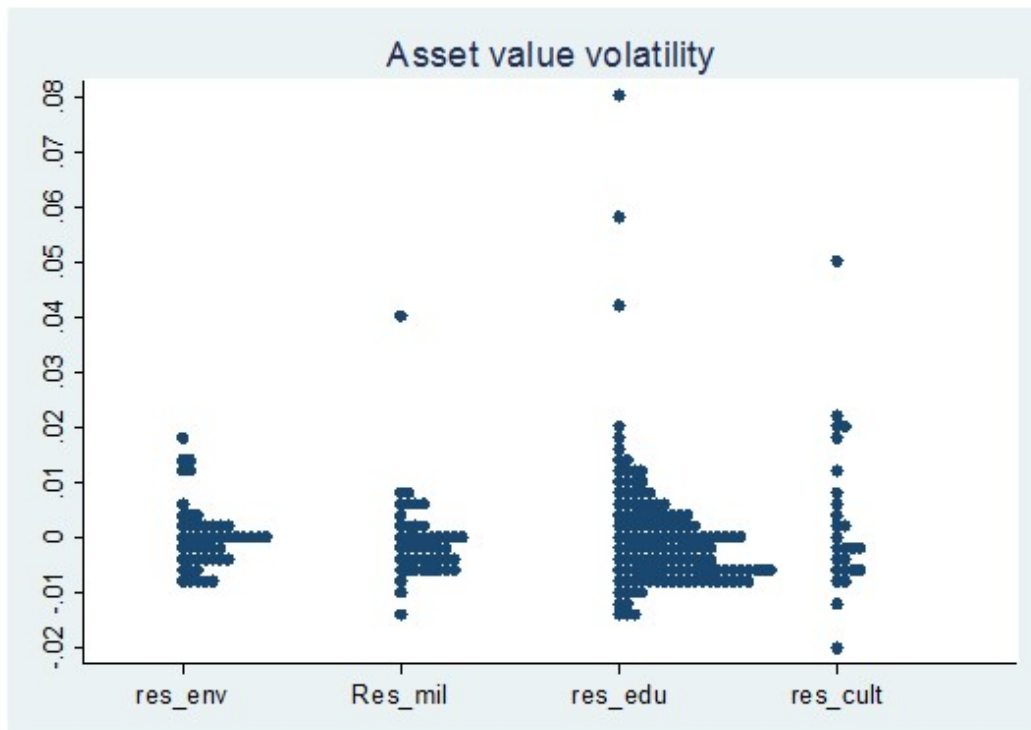
for respective facility sectors are summarized in the below table:

Table 10. Unexpected volatility of asset value

Sectors	$E(\frac{\Delta of Asset Value_j}{Asset value_i})$	$\delta(\frac{\Delta of Asset Value_j}{Asset value_i})$
Environment	0.0001958	0.00586
Military	-0.0006510	0.00762
Education	-0.0002595	0.01142
Culture	0.0028926	0.01411

The means of expected deviations in all sectors are not significantly different from zero based on t-value test (please refer to the appendix 7). It means the estimated asset value volatility is eligible for Merton's option model as it assumes a random walk of asset value volatility.

Figure 11. Distributions of unexpected volatility of asset values



The input assumptions for calculating coupon spread (${}_0CS_t$), using Merton's option model are summarized as below table:

Table 11. Assumptions for estimating risk spread of operation period

Asset value	Asset value _(t=0) = Bond value _(t=0) = 1.00 (100% LTV ratio in ex-ante basis)
Bond value	
Duration (t)	1 ~ 20 years
Asset value volatility $\delta(\frac{\Delta of Asset Value_j}{Asset value_i})$	Environment : 0.00586 Military : 0.00762 Education : 0.01142 Culture : 0.01411
Risk free rate (R _f)	Environment : 4.563% Military : 4.760% Education : 4.946% Culture : 4.697%

The 20 coupon spreads (${}_0cs_t$) with different maturities from one to twenty years are calculated applying above assumptions to Merton's option model. The coupon spreads (${}_0cs_t$) for each of maturities are estimated in four separate sectors.

Table 12. Estimated coupon spread with maturity 't' (${}_0cs_t$)

maturity (t)	environment ($\delta = 0.00586$)	military ($\delta = 0.00762$)	education ($\delta = 0.01142$)	culture ($\delta = 0.01411$)
1st coupon	0.2341%	0.3045%	0.4565%	0.5644%
2nd coupon	0.1656%	0.2155%	0.3231%	0.3996%
3rd coupon	0.1353%	0.1760%	0.2640%	0.3265%
4th coupon	0.1172%	0.1525%	0.2288%	0.2830%
5th coupon	0.1049%	0.1364%	0.2047%	0.2533%
6th coupon	0.0957%	0.1246%	0.1870%	0.2314%
7th coupon	0.0887%	0.1154%	0.1732%	0.2143%
8th coupon	0.0830%	0.1080%	0.1621%	0.2006%
9th coupon	0.0782%	0.1018%	0.1529%	0.1892%
10th coupon	0.0742%	0.0966%	0.1451%	0.1796%
11th coupon	0.0708%	0.0921%	0.1384%	0.1713%
12th coupon	0.0678%	0.0882%	0.1325%	0.1641%
13th coupon	0.0651%	0.0848%	0.1274%	0.1577%
14th coupon	0.0628%	0.0817%	0.1228%	0.1520%

15th coupon	0.0607%	0.0790%	0.1187%	0.1469%
16th coupon	0.0587%	0.0765%	0.1149%	0.1423%
17th coupon	0.0570%	0.0742%	0.1115%	0.1381%
18th coupon	0.0554%	0.0721%	0.1084%	0.1343%
19th coupon	0.0539%	0.0702%	0.1055%	0.1307%
20th coupon	0.0526%	0.0685%	0.1029%	0.1274%

The optimal risk spread of the whole operation period ($rs^*_{o,t}$) is calculated based on the time weighted average of estimated coupon spreads (${}_0cs_t$).

Table 13. Optimal BTL risk spread in operation periods ($rs^*_{o,t}$)

t	<i>t × coupon spread (${}_0cs_t$)</i>			
	environment	military	education	culture
1	0.2341%	0.3045%	0.4565%	0.5644%
2	0.3313%	0.4309%	0.6463%	0.7991%
3	0.4059%	0.5280%	0.7921%	0.9796%
4	0.4688%	0.6100%	0.9152%	1.1320%
5	0.5243%	0.6822%	1.0237%	1.2665%
6	0.5745%	0.7476%	1.1220%	1.3882%
7	0.6207%	0.8077%	1.2124%	1.5002%
8	0.6637%	0.8637%	1.2967%	1.6047%
9	0.7041%	0.9164%	1.3759%	1.7028%
10	0.7423%	0.9662%	1.4508%	1.7957%
11	0.7786%	1.0136%	1.5222%	1.8842%
12	0.8134%	1.0589%	1.5904%	1.9688%
13	0.8468%	1.1023%	1.6559%	2.0500%
14	0.8789%	1.1442%	1.7189%	2.1282%
15	0.9099%	1.1846%	1.7798%	2.2037%
16	0.9398%	1.2237%	1.8387%	2.2768%
17	0.9689%	1.2616%	1.8958%	2.3477%
18	0.9971%	1.2984%	1.9513%	2.4165%

19	1.0246%	1.3342%	2.0053%	2.4836%
20	1.0513%	1.3691%	2.0579%	2.5489%
Sum	14.4789%	18.8478%	28.3076%	35.0417%
$rs_{o,t}^* = \frac{\sum_{t=1}^T cs_t \times t}{\sum_{t=1}^T t}$	0.0689%	0.0898%	0.1348%	0.1669%

On top of the estimated risk spread, risk free rates (R_f)⁴⁰⁾ and a long-term investment premium (refer to appendix 3) are added to get the optimal return for BTL projects in operation period ($r_{o,t}^*$) as below table:

Table 14. Optimal BTL risk spread ($rs_{o,t}^*$) and return ($r_{o,t}^*$) in operation period

Sectors	risk spread in operation period ($rs_{o,t}^*$)	risk free rate (R_f)	long term premium	optimal return in operation period ($r_{o,t}^*$)
Environment	0.0690%	4.5632%	0.457%	5.0892%
Military	0.0898%	4.7603%		5.3071%
Education	0.1349%	4.9457%		5.5376%
Culture	0.1670%	4.6970%		5.3210%

40) The risk free rates are estimated in a consistent way to optimal return in construction period: all of the five years sovereign bond rates at respective contract dates are collected and averaged in each of sectors

3. The optimal return (r_t^*) and risk spread (α^*) of BTL projects

The optimal level of life-cycle BTL investment return (r_t^*) is estimated by Θ weighted average based on the optimal returns in construction period ($r_{c,t}^*$) and operation period ($r_{o,t}^*$). The weighting factor ' Θ ' is the share of return for construction period in total. The derivation of ' Θ ' is shown in (appendix 8).

$$r_t^* = \theta \cdot r_{c,t}^* + (1-\theta) \cdot r_{o,t}^* \quad (4.19)$$

r_t^* : optimal return for BTL project assessed at time 't' (beginning of project)

$r_{c,t}^*$: optimal rate of return for construction period

$r_{o,t}^*$: optimal rate of rerun for operation period

Θ : share of return for construction period in total

Table 15. Optimal BTL return for the life cycle project period

sectors	$r_{c,t}^*$	$r_{o,t}^*$	θ	r_t^*	$\bar{r}_{contract}^{41)}$
environment	7.6726%	5.0892%	0.1291	5.423%	5.61%
military	8.2680%	5.3071%	0.1289	5.689%	5.97%
education	7.7424%	5.5376%	0.1293	5.823%	6.09%
culture	7.8676%	5.3210%	0.1291	5.650%	6.05%

Finally, the optimal risk spread (α^*) of BTL is calculated by subtracting risk free rates (R_f) from the above optimal project returns (r_t^*) of BTL. Now, the estimated optimal risk spread (α^*) is fairly comparable with historical contract alpha (α).

Table 16. Optimal BTL risk spread (α^*) and contract average ($\bar{\alpha}_{contract}$)

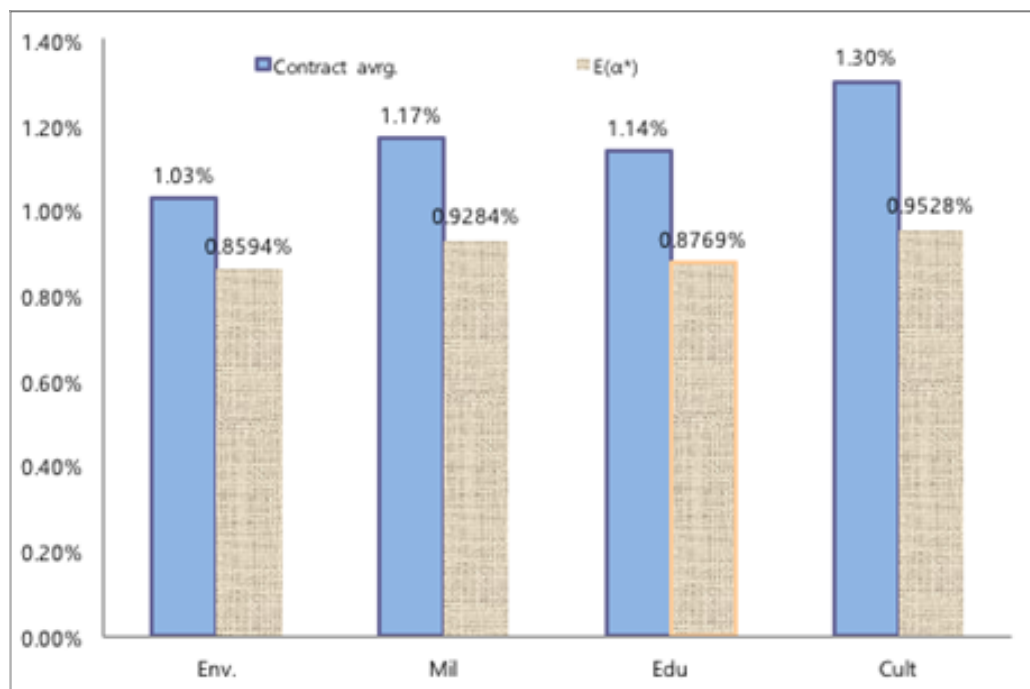
sectors	r_t^*	R_f	$\alpha^* (= r_t^* - R_f)$	$\bar{\alpha}_{contract}$
environment	5.423%	4.5632%	0.8594%	1.03%

41) Track record of average contract returns ($\bar{r}_{contract}$) are summarized for reference purpose.

military	5.689%	4.7603%	0.9284%	1.17%
education	5.823%	4.9457%	0.8769%	1.14%
culture	5.650%	4.6970%	0.9528%	1.30%

The optimal average risk spread (α^*) of environment sector is estimated as 0.859% and the historical average alpha of 96 contracts is 1.03%. The optimal spread of military sector is estimated as 0.928% and the historical average of 70 military contracts is 1.17%. The optimal spread of education sector is 0.877% and the historical average of 218 education contracts is 1.14%. The optimal average risk spread of culture sector is measured as 0.953%, but the average track-record spread of 42 culture facilities is 1.301%, which is far higher than the estimation result of optimal level.

Figure 12. Contract average and optimal level of risk spread (α^*)



Intuitively speaking, the physical complexity of product and expected degree of deviation in construction and operating costs are reflected into the different levels of

optimal alpha. Examples of the environment facilities are sewage systems that are networks of pipelines buried under ground, causing the lowest operation risk. Whereas, examples of military facilities are simple structured town houses or apartments. Most of education facilities are schools including several story buildings and additional structures such as dining room, auditorium, playground or gym etc. The optimal return of military sector is slightly higher than that of education sector due to the higher average beta of construction sponsors but the gap is not significant. Culture facilities have to meet diverse visitor's requirements, and each building needs unique design to fulfill the purpose of cultural need, causing the highest operation risk. It can be maintained that the required risk premium of BTL facility implicitly reflects the degree of expected risk for construction and volatility of net operating income.

4. Hypothesis test

Based on the above study on optimal risk spread, the null hypothesis (H_0) is tested to draw a conclusion.

$$H_0 : \bar{\alpha}_{contract} - \alpha^* = 0$$

$$H_a : \bar{\alpha}_{contract} - \alpha^* \neq 0$$

$\bar{\alpha}_{contract}$: average contract BTL risk spread of Korea
 α^* : optimal level of BTL risk spread

The null hypothesis is 'the historical average of BTL contract spread (alpha) is statistically not different level from the optimal risk spread in Korea'. And the alternative hypothesis is 'there is no evidence that the historical average of BTL contract spread (alpha) is statistically not different level from the optimal risk spread in Korea'. The track record of average and standard deviations of risk spread (α) in four groups of BTL contracts are summarized in the below table:

Table 17. Historical alpha and optimal level estimation result

statistics	environment	military	education	culture
contract average ($\bar{\alpha}_{contract}$)	1.027%	1.166%	1.138%	1.301%
standard deviation of alpha	0.00177	0.00127	0.00188	0.00325
number of observations	96	70	218	42
optimal alpha (α^*)	0.8594%	0.9284%	0.8769%	0.9528%

Two-tailed z-test is performed. If the calculated z-value in absolute number is bigger than the critical value, the null hypothesis should be rejected. The z-values of four facility groups are calculated based on the below formula and compared with the critical value with 99% of confidence level. The hypothesis test results are summarized in the below table:

$$Z = \frac{[\alpha_{contract} - \alpha^*]}{[\frac{\delta(\alpha)}{\sqrt{n-1}}]} \quad (4.20)$$

Where, $|C.V(Z_{99\%})| = |\pm 2.576| = 2.576$

Table 18. The hypothesis test result

test	environment	military	education	culture
z - values	9.0757	15.1842	20.2418	6.7570
Comparison against critical value	$Z > C.V(Z_{99\%})$	$Z > C.V(Z_{99\%})$	$Z > C.V(Z_{99\%})$	$Z > C.V(Z_{99\%})$
Hypothesis : H_0	Rejected with 99% confidence	Rejected with 99% confidence	Rejected with 99% confidence	Rejected with 99% confidence

The null hypothesis is rejected in all sectors under 99% of confidence level. The test result reveals that the average contract risk spread (alpha) in all sectors are above optimal level, maybe influenced by additional idiosyncratic factors. As a result, there is no evidence that BTL market of Korea is fully efficient. The degree of excess spread of alpha are different among groups: the excess spread of alpha in environment sector is 0.1677%p, military sector is 0.2380%p, education sector is 0.2619%p, and culture sector is 0.3482%p. It is concluded that, as is usual, there exist inefficiency in BTL market of Korea but the level of excess returns are different among sectors.

One thing that should be mentioned is the higher level of contract alpha than the optimal level does not necessarily mean a mis-pricing. The historical contract alpha must have been determined reflecting both systematic and part of idiosyncratic risks. A considerable portion of idiosyncratic risk may not be fully removed by diversification in the real world, especially in infrastructure market. As a result, if market is not fully efficient, the optimal risk spread is conceptually different from a

fair market price which should be determined including idiosyncratic risk that is unique in infrastructure market and thus individual investor can not remove any more even with best effort.

However, considering the existence of excess return, it may be said that policy makers has a chance to make efforts for enhancing market efficiency and controlling BTL return within affordable level in overall. One of such effort for reducing the gap between contract and optimal alpha may be a rigorous project assessment using a lower hurdle rate. Korean government tried to control average BTL risk spread within Nationally affordable level from the commencement of BTL in 2005. The ex-ante project assessment is performed to secure value-for-money (Vfm), where the 6% of discount rate is currently used, meaning the hurdle of investor's IRR is allowed up to 6%. As the 6% of hurdle rate is higher than the optimal level, some of the projects may exploit excess return, although not substantial. The above (table 15) compares the optimal level of BTL return with the average contract record in each sector. The average contract return of environment sector is 0.19%p, military sector is 0.28%p, education sector is 0.27%p, and that of culture sector is 0.40%p higher than the optimal returns. Based on the study result, optimal level of average BTL return reflecting systematic risk is around 5.4% ~ 5.8%, which result is below the current 6% of hurdle rate.

In conclusion, the result of hypothesis test indicates that there remains a room for enhancing public benefit by improving market inefficiency and decreasing project return. Probably, there will exist different levels of idiosyncratic factors in respective sectors. The following chapter empirically investigates what idiosyncratic factors influenced how much impact on alpha to draw policy implications and conclusion.

V. Idiosyncratic determinants of BTL project return

Systematic risk is caused by market structure or dynamics which produce uncertainty faced by all companies in the market. In contrast, idiosyncratic⁴²⁾ risk is uncertainty to which only specific company or industries are vulnerable, and it is uncorrelated with broad market returns. Due to the idiosyncratic nature of unsystematic risk, it can be reduced or eliminated through diversification however, since all market actors are vulnerable to systematic risk, it cannot be hedged through diversification.

The chapter 'IV. Optimal BTL risk premium for BTL projects' searched the optimal level of risk spread for BTL projects from the theoretical point of view and compared it with historical average. It is assumed that investors can constitute a fully diversified portfolio to achieve an efficient invest line by getting rid of unsystematic risk. As a result, the optimal project return is considered as a compensation for systematic risk that can't be reduced any more. It means the theory assumes an efficient equilibrium market that is hard to exist in the real world, especially in PPP market. And thus, idiosyncratic factors that additionally influenced on contract return are not considered in the theoretical level of risk spread, but they may explain the discrepancies between theoretical and actual spreads.

In practice, the contract return is determined through negotiation and benchmarking to former comparable projects. Of course, the contract return of infrastructure reflects various factors that are unique to specific projects. The infrastructure market is known as one of the most inefficient possibly due to information asymmetry, mega size capital requirement, and multi-disciplinary expertise requirement, etc. And that, most of the local SPCs are unlisted, which business data are hard to be accessed by public. The core idea of PPP is sharing risk between public and private parties and thus, the degree of shared risk to private party changes depending on the risk allocation clause of

⁴²⁾ The term of 'idiosyncratic' is used interchangeably with 'unsystematic' in this research

contract. As a result, contract terms or policy directions at contract point can make a severe influence on the required rate of return.

It being said, the empirical analysis is additionally performed to fully understand idiosyncratic factors that affected on contract return, degree of influence, and to draw policy considerations. The categories of candidate factors are macro financial market, unique characteristics of individual project, PPP policy, and the degree of market maturity, etc. Through the empirical analysis in this chapter, we expect to get additional understandings on the determinants of contract return in practice, and may be able to draw some political implication for further improvement of public good.

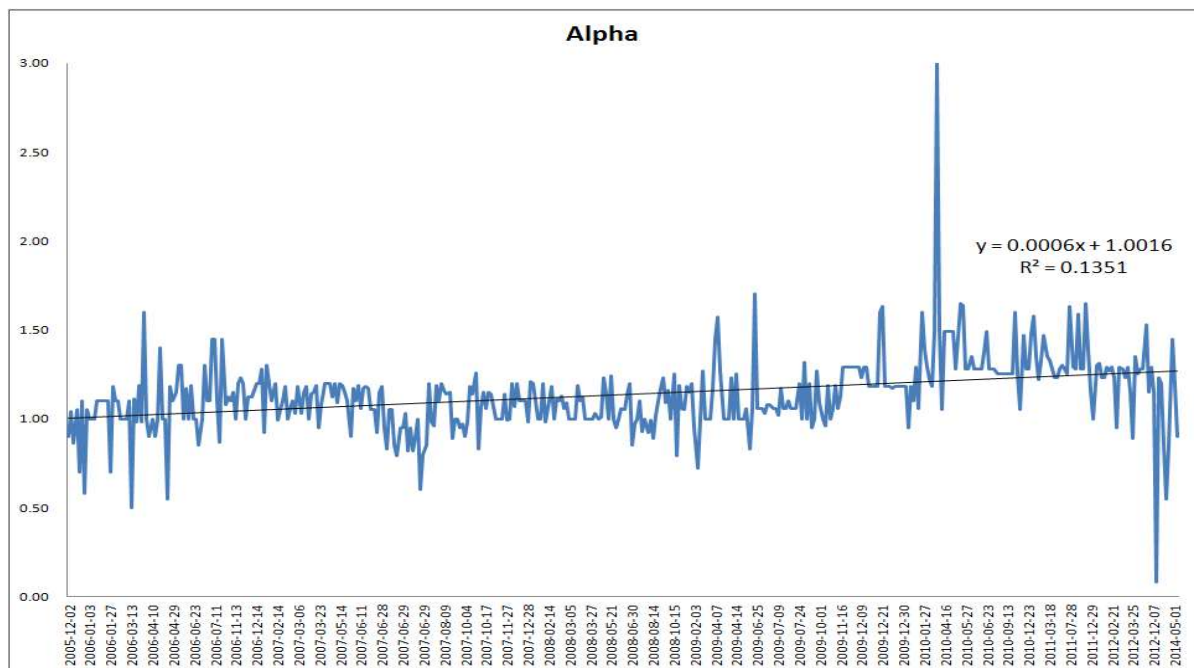
1. Historical profit level of BTL contracts

Most of the social infrastructures that do not generate enough cash to compensate capital investment are built through BTL in Korea. Examples are environment facilities, military bases, schools, and culture facilities. The average return of 426 BTL contracts from 1995 to 2014 is 5.96%, ranging from 3.46% to 8.48%. The project returns in the early period was 5~6%. It gradually increased up to 6~7% level until 2009, and again decreased down to around 4% level in 2010. The recent contracts even show mid 3% of return due to the dramatic depression of interest rate in Korea.

The contract return is comprised of risk spread (alpha) and five years variable sovereign bond rate, which is considered as a proxy to risk free rate. The risk spread (alpha) is determined by contract and shall not be changed throughout concession period. As a result, the meaningful milestone that really determines the profit of project is the risk spread (so called, alpha) in BTL. The average alpha of 426 local BTL contracts from 1995 to 2014 is 1.13%, ranging from 0.03% to 3%. The time-series contract risk spread (alpha) didn't show a trend, having been very flat. However, in 2014, it has dropped down to about 1% level due to the depressed financial market and hard competition in local market. The time-series trend and distribution of alpha

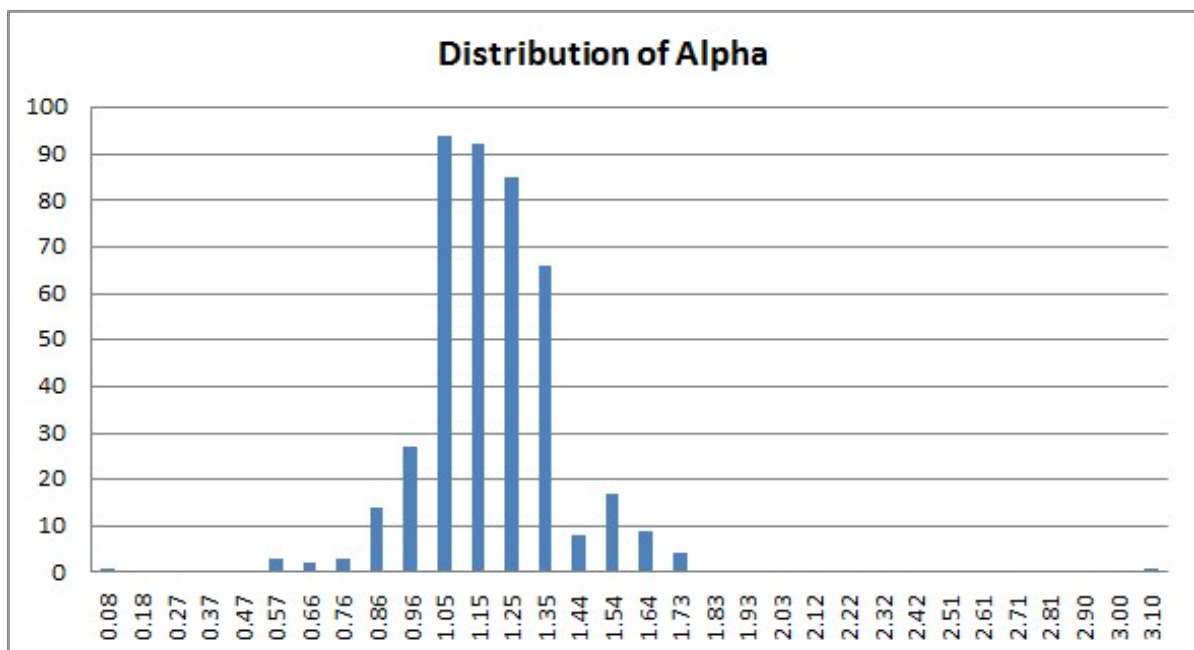
are as below:

Figure 13. Time series trend of contract alpha



(Source: Comprehensive PPP evaluation, PIMAC)

Figure 14. Distribution of historical contract alpha



(Source: Comprehensive PPP evaluation, PIMAC)

The recognized investment risk of the four major sectors of facilities: environment, military, education, and culture facilities are probably different to investors, and so the average risk spread (alpha) for each category shall be different. The statistics of contract alpha in four sectors are summarized in the below table:

Table 19. Project return and alpha statistics

(unit, %)

statistics		Minimum	Average	Maximum
Environment (96 projects)	Project IRR	3.46	5.61	6.81
	Base rate ⁴³⁾	2.83	4.59	5.98
	Risk spread (alpha)	0.5	1.03	1.38
Military (70 projects)	Project IRR	4.89	5.97	8.48
	Base rate	3.97	4.81	7.39
	Risk spread (alpha)	0.79	1.17	1.29
Education (218 projects)	Project IRR	3.61	6.09	7.68
	Base rate	2.93	4.96	6.12
	Risk spread (alpha)	0.08	1.14	1.70
Culture (42 projects)	Project IRR	4.11	6.05	7.17
	Base rate	2.79	4.75	5.83
	Risk spread (alpha)	0.86	1.30	3.00

(Source : Vfm assessment guideline for BTL PIMAC)

As is discussed, the alpha spreads of the four categories don't show clear time-series trends, having a steady level respectively for each category. However, the alpha showed a bit inverse proportional to base (five years bond) rate because the base rate continually decreased while the level of alpha increased a bit slowly. An empirical analysis through OLS is performed to study what factors influenced how much impact on the risk spread alpha on top of the systematic risk factors.

⁴³⁾ Five days average of five years sovereign bond rate at each of the contract dates

2. Empirical analysis

The four categories of candidate variables that may explain the level of BTL risk spread are selected as follows:

Table 20. Candidate determinants of risk spread (alpha)

Category	Determinants	Exogenous Variable
Financial market	risk free rate (rf)	five years sovereign bond rate at individual contract point
	market risk spread (aa_rf)	AA ⁻ corporate bond rate minus five years sovereign bond rate at individual contract point
Project characteristics	funding cost spread (kd_spread)	total debt funding cost minus base rate (AA ⁻ corporate bond) at individual contract point
	facility type (milit, edu, cult,)	environment facilities being base case, military, education, and culture facilities are assigned as binomial variables to stratify facility types.
	project size (invest_size)	natural log of investment amount in million won
	subsidy grant (sub_binom)	binomial variable: one is assigned if subsidy is granted, and zero is assigned if subsidy is not granted
Procuring process	procuring authority (authority)	binomial variable: central ministries being base case, one is assigned to municipal governments
	preparation period (period)	preparation and negotiation duration from Request for Proposal (RFP) announcement to contract date (in days)
Market maturity	bidding ratio (bid_ratio)	number of bidders joined preferred bidder selection process

The first category is the financial market factors. They are definitely not idiosyncratic factors but systematic market factors. The reason that market factors are included in the analysis is a control purpose. If these conditions are not controlled, we can not fairly compare the impact of idiosyncratic variables in a equal condition. For example, if one contract is made when risk free rate is 5% and the other is made when risk free rate is 8%, the starting point of alpha will be different from the

beginning, and the degree of impact due to idiosyncratic factors will probably be misled. As a representative risk free rate (R_f) in market factor, the five years sovereign bond rate is selected. The market risk spread (aa_Rf) is presented by risk spread between AA⁻ corporate bond and risk free rate that is consistently measured by five years sovereign bond rate. A concern may be raised that a possible correlation between risk free rate (R_f) and market risk spread (aa_Rf) may cause a ‘multicollinearity’ issue to the regression models. So, similar regression tests are performed excluding the risk free rate variable to check if the result is significantly changed (please refer to appendix 8).

The second category is idiosyncratic factors that are unique to individual projects or sectors. The funding cost spread (kd_spread) represents the cost of debt funding in project level. To reduce a possibility of correlation with market factors, the risk spread of individual project funding is selected as variable. The risk spread of individual project funding, which should be able to show the difference of funding cost caused by individual project risk and sponsor’s credit, is calculated by total funding cost minus AA⁻ bond rate at each of contract points. It is expected that if a funding cost of individual project increases, the required alpha increases, too. The project types (milit, edu, cult) are binomial variables that stratify the average difference of alpha among sectors. It is highly possible that investors will require different levels of alpha depending on forecasted levels of construction and operation risk. The project size ($invest_size$) may affect to the level of alpha maybe due to the risk averseness of human being or economy of scale. In case the former factor is stronger, the correlation coefficient of this variable will be positive but if the latter factor is stronger, the result can be negative. The subsidy (sub_binom) is a binomial variable that one is assigned when subsidy is granted and zero is assigned if not. This variable is selected to see if significant difference occurs in alpha due to a provision of subsidy. A subsidy has a smaller importance in BTL compared with BTO, where it is granted to make a project financially feasible in case project does not meet minimum level of required IRR. Most of the transportation infrastructures

are lack of commercial viability due to the low level of user fees in many economies. By injecting some amount of construction subsidy, the capital investment of private party decreases, resulting in enhanced IRR of private investors if the forecasted revenue is same. Anyway, there is no concern for financial feasibility in BTL because project IRR is pre-determined by the level of government's installment payment, and thus there is little motivation for providing subsidy in BTL. That's why only 22 numbers of projects among 426 of total population have construction subsidy.

The procuring authority (authority) and project preparation period (period) are variables related with preparation process. The procuring authorities may have different criteria in determining alpha. So, they are categorized as central ministry which is assigned zero, and municipal governments which are assigned one. The project preparation period is measured by the number of days from RFP announcement to contract date of each project. It is expected if a project is prepared for longer period with enough negotiation, the alpha may be lower than others.

The market maturity is measured by market competition: the historical bidding ratio (bid_ratio) in preferred bidder selection process. It is expected that as market gets matured, the degree of competition will increase, and as a result, the level of alpha may decrease.

Using above factors as exogenous and contract risk spread (alpha) as endogenous variables, a regression analysis is performed. The degree of influence can be statistically analyzed by translating the result of correlation coefficients and standard errors. The 426 contract data are used for the analysis. The regression results of three models are summarized as below:

Table 21. Regression result of alpha on determinants

VARIABLES	(1) alpha	(2) alpha	(3) alpha
rf	-0.0877*** (0.0150)	-0.0871*** (0.0146)	-0.0893*** (0.0148)

aa_rf	-0.0599*** (0.0127)	-0.0601*** (0.0126)	-0.0613*** (0.0126)
kd_spread	0.186*** (0.0267)	0.187*** (0.0265)	0.207*** (0.0264)
invest_size	-0.0428** (0.0173)	-0.0413** (0.0168)	-0.0706*** (0.0147)
sub_binom	0.0376 (0.0395)		
bid_ratio	-0.0213** (0.0100)	-0.0214** (0.00987)	-0.0236** (0.00997)
authority	-0.0175 (0.0411)		
period	1.21e-05 (6.51e-05)		
milit	0.0727 (0.0514)	0.0906*** (0.0331)	
edu	0.0902*** (0.0295)	0.0928*** (0.0270)	
cult	0.173*** (0.0411)	0.182*** (0.0378)	
Constant	1.775*** (0.222)	1.743*** (0.210)	2.128*** (0.189)
Observations	329	329	329
R-squared	0.395	0.393	0.347

Robust standard errors in parentheses

*** p<0.01, **p<0.05, * p<0.1

The above table shows three steps of trial to find out the fittest one, which is model(2). All of the candidate determinants are included in the first model, from which variables that don't meet confidence level of 90% are excluded in model(2). From the first and second trial, it is concluded there is no evidence that the three variables: 'sub_binom', 'authority' and 'period' do not meet 90% of confident level and thus, they are excluded. As a result, most of the correlation coefficients of remaining variables in model(2) meet over 95% of confidence level. And all of the remaining variables in model(3) show p-values that are small enough even though sector dummies are additionally pulled out. It also provides an evidence of robustness. The correlation coefficient of constant is statistically significant in all cases. As the main purpose of the empirical analysis was to understand the impact of idiosyncratic

factors on alpha, the constant was out of interest when designing the test. However, it may be said that the average alpha of BTL is 2.128%, assuming all the other variables such as risk free rate, market risk spread, interest cost spread, investment size, and bidding ratio be zero.

(1) Financial market

In the result of second regression model, the correlation coefficient of 'risk free rate (rf)' is estimated as (-)0.0871, which result means if other conditions are equal and the risk free rate increases 1%p, the average alpha will decrease 0.0871%p. That of 'market risk spread (aa_rf)' is estimated as (-)0.0601, which result also means if other conditions are equal and the the market risk spread increases 1%p, the alpha will decrease 0.0601%p in average.

The results are consistent with the observation that market interest rate gradually decreased, while alpha spread increased slowly for a decade in the local market. It can be explained by the market competition of Korea and long-term investment nature of PPP. In practice, investors recognize BTL as a long-term business and thus they are tend to maintain total IRR within bankable range. If base rate decreases, investors tend to increase alpha to compensate long-term IRR within limited scope. In the other way round, if base rate increases, investors are willing to decrease alpha to win a bid within limited scale. If investors decrease alpha too much, they will be open to ruinous long-term interest rate risk when base rate decreases. This result provides an empirical evidence that the total return of BTL has some degree of rigidity against market interest rate. And it strengthens the observation that investors tend to look for a long-term spread and thus, temporary decrease of market rate makes them increase alpha in some degree to secure the total investment return over minimum investable level of total return, and vice versa if market rate increases.

One possible concern in the analysis may be a correlation between risk free rate

(Rf) and market risk spread (aa_Rf). Which relation may cause a ‘multicollinearity’ issue to the regression models. So, similar regression tests are performed excluding the risk free rate (Rf) variable. Small changes occurred in the analysis results however, most of the implications and translations are kept same as the above original models. The test result excluding risk free rate variable is added in appendix. (please refer to the appendix 9)

(2) Project characteristics

As is explained, the nature of BTL investment is similar to buying a sovereign bond that has additional risk of construction and operation. The net cash flow of equity investor is the project return minus debt funding (interest) cost. As a result, the project return should be able to cover required minimum return on equity plus debt funding cost, which is generally determined by three years AA⁻ bond rate plus funding cost spread⁴⁴⁾ in the local PF market. In summary, alpha represents interest revenue whereas, beta represents interest cost from the viewpoint of equity holders in BTL. So, investors will bid alpha, which level is able to cover both the funding cost spread and equity holder’s target profit. Of course, the funding cost spread is priced in project financing (PF) market based on project risk and investor’s credit ratings, both of which are given condition to investors. It being said, it is natural if funding cost spread level increases alpha should increase, too. In the above empirical analysis result, the correlation coefficient of the ‘funding cost spread (aa_rf)’ showed 0.187, which is one of the strongest exogenous variables. It is concluded that, if other conditions being equal and the funding cost spread (kd_spread) increases 1%p, the average alpha increases 0.187%p under 99% of confidence. It is found that the funding cost has a strong impact on the contract alpha. One of the reasons that the average risk spread of environment sector could be lower than other sectors is the

44) The ‘funding cost spread’ here is the interest rate spread over base rate of ‘funding’ cost. It does not have relationship with the risk spread (alpha) for investors in BTL projects.

lowest average funding cost spread. The average funding cost spread in environment sector is 1.3267%, which is the lowest level, compared with other sectors: 1.5267% in military, 1.4073% in education, and 1.6600% in culture. In summary, the lowest funding cost of environmental facility sector contributed to forming the average market risk spread (alpha) in the lowest level.

The ‘investment size (invest_size)’ variable has the correlation coefficient of (-)0.0413, which result means if other conditions are equal and the investment amount increases by \$10⁶e, the average alpha will decrease 0.0413%p under 99% confidence level. Regardless of the statistical confidence level, the value of correlation coefficient is too small to say investment size has meaningful impact to alpha, considering the unit amount of independent variable is natural log of million US dollars. It can be said that a project size does not have meaningful influence on the contract risk spread (alpha), which fact may suggest two things. First of all, BTL investors don’t require increased marginal return for bigger investment size. It may be caused by the risk profile of BTL, which most investors recognize as similar to buying a sovereign bond after the subject facility is constructed. This observation also supports the application of bond valuation approach, the Merton’s option model in this research. The impact of size variable can also explain why bundling of BTL projects such as schools or dormitories are popular and effective. Second guess is that BTL projects are small sized compared with BTO projects and the difference of size among projects is not significant. So, the marginal impact of investment value to alpha spread may have been limited. When the binomial variables that stratify sectors are excluded in (model 3), it is observed that the absolute value of correlation coefficient of size variable increases. This means the impact of size deviation across sectors are bigger than the deviation within a sector.

The ‘subsidy (sub_binom)’ variable shows correlation coefficient that is not statistically reliable. As is expected, the construction subsidy has almost no relationship with the level of contract alpha. In general, the construction subsidy is

granted to boost project return in case a project is financially not feasible due to a low demand in BTO. However, in BTL project, as procuring authority pays fixed amount of installment payments, there remains no demand risk to private investors. One of the reason that procuring authority grants subsidy in BTL is to allocate government's burden between present and future. That is why construction subsidy is granted to only small portion of samples (22 projects among 426 of total population).

The correlation coefficients of dummy variables such as ‘milt’, ‘edu’, ‘cult’ indicate the difference of average alpha spread among sectors, controlling other conditions being equal. On the basis of regression analysis result, historical BTL investors in military sector required 0.091%p higher risk spread than the investors in environment sector under 99% confidence level. In the same way, the investors in education sector required 0.093%p higher, and those in culture sector required 0.182%p higher risk spread than environment sector under 99% confidence level.

Comparing this result with the optimal level of alpha, the stratified differences among sectors by regression analysis is bigger. For example, the average risk spread of military group is 0.091%p higher than that of environment sector based on the regression analysis, while the gap of risk spreads between environment and military sectors is only 0.069% based on the optimal alpha. The below table shows the gaps of average alpha among sectors based on empirical regression analysis and those from optimal level estimation.

Table 22. Difference of average risk spread over environment sector

methodology		environment	military	education	culture
empirical regression analysis		N/A	0.091%p	0.093%p	0.182%p
estimated optimal	optimal alpha	0.8594%	0.9284%	0.8769%	0.9528%
	gap over environment sector	N/A	0.069%p	0.018%p	0.093%p

However, the above gaps among groups can not fairly compared. The stratified

difference of spreads among groups in the first line of above table is calculated, with other conditions controlled. However, the difference of average alpha among groups in the third line is calculated based on the optimal alpha that all other different conditions are reflected. For this reason, additional test is performed to fairly compare the degree of difference in average alpha among sectors based on the regression line (model 3). The regression line can be expressed as below formula:

$$\alpha_i = 2.128 - 0.0893 \times Rf - 0.0613 \times aa_rf + 0.207 \times kd_spread - 0.0706 \times invest_size - 0.0236 \times bid_ratio + r_i$$

(0.189) (0.0148) (0.0126) (0.0264) (0.0147) (0.00997)

And the alpha of all projects are estimated by substituting historical data to all the variables in the below line:

$$E(\alpha_i) = 2.128 - 0.0893 \times Rf - 0.0613 \times aa_rf + 0.207 \times kd_spread - 0.0706 \times invest_size - 0.0236 \times bid_ratio$$

(0.189) (0.0148) (0.0126) (0.0264) (0.0147) (0.00997)

The residual value of alpha in all projects are calculated ($r_i = \alpha_i - E(\alpha_i)$), and the distribution of residual values (r_i) in respective sectors are measured as below table:

Table 23. Distribution of estimation errors (r_i) in four sectors

r_i	environment	military	education	culture
mean	-0.0735%	-0.0122%	-0.0037%	0.0754%
SSE	0.0329	0.0104	0.0319	0.0560
gap of means over environment	N/A	0.061%p	0.070%p	0.150%p

The means of residual value in above table represent the degree of gap from average market line for alpha of each sector. As such, the third line represents the gap of each sector's average alpha over environment sector. The mean of environment is negative and that of culture is positive, which means the average contract alpha of environment sector exists below the average market line and that of culture sector is

located above the line.

Table 24. Gap of optimal alpha among sectors

estimation result	Environment	Military	Education	Culture
Optimal alpha	0.8594%	0.9284%	0.8769%	0.9528%
spread over environment sector	N/A	0.069%p	0.018%p	0.093%p

Now, the different levels of average alpha among sectors can be fairly compared. The gap of military over environment sector in (table 23) is about 0.061%, which is close to the gap of optimal alpha, 0.069% in (table 24). However, education and culture sectors have higher gaps over environment sector in the regression analysis based on contract record. It confirms that investors in education and culture sectors required higher excess returns implicitly recognizing additional idiosyncratic factors. This test shows different levels of excess returns exist depends on sectors.

(3) Procuring process and Market maturity

The ‘procuring authority (authority)’ variable is included to test whether a difference in procuring authority may cause an influence on the contract alpha in BTL. The correlation coefficient of 'authority' variable is not reliable under 90% of confidence level. In conclusion, there is no evidence that risk spread (alpha) has meaningful difference due to procuring authority.

The ‘preparation period (period)’ variable is included to test how much impact the project preparation period (in days) has on the contract alpha. From the above analysis result, it is concluded that there is no statistical evidence that project preparation period influences on the project spread, alpha.

The variable, ‘bid_ratio’ represents the number of bidders who joined the preferred

bidder selection. The correlation coefficient of bid_ratio variable is (-)0.0214, which means the alpha spread decreases 0.0214%p as one additional bidder joins. This variable has a unique characteristics, different from other variables, in that market competition can contribute to public good through increased efficiency. It is concluded that the degree of PPP market competition (or maturity) can contribute to public good by decreasing required rate of return for private investors. One of the reasons that the average alpha of environment sector is formed to the lowest level may be the highest average bid ratio, which is 2.270 (that of military is 1.710, that of education sector is 2.100, and that of culture sector is 2.020). Compared with others, the culture sector has the smallest number of BTL track record⁴⁵⁾ and so, it can be said that the BTL market of culture sector has the lowest maturity than other groups. It may be able to explain part of reason why the excess return of culture is exceptionally higher than any other sectors.

45) Total population of culture group is only 42, which is far smaller than other groups: 96 in environment, 70 in military, and 218 in education.

VI. Policy implication

The hypothesis test in chapter IV indicates that the average contract risk spread is different from the optimal level. And it is concluded that there remains a room for enhancing public benefit by improving market inefficiency and decreasing project return. The empirical test of the chapter V reveals the funding cost has the strongest impact on the contract alpha. For example, the lowest average funding cost of environment sector contributed to forming the lowest average contract alpha. Another reason for low level of alpha in environment sector is the highest bid ratio, which is 2.270⁴⁶⁾. It is also found that the BTL market of culture sector has the lowest market competition as well as the lowest maturity than other groups, resulting in the highest level of deviation of average contract alpha from the optimal level of risk spread. The distribution of residual values measured from regression line (model 3) also supports the result of hypothesis test in chapter IV that different levels of excess returns exist among groups. The stratified differences of average spreads in military, education, and culture sectors over environment are bigger than the gaps of optimal alpha levels among sectors. This result indicates the investors required higher excess spreads in education and culture sectors than environment, implicitly recognizing additional idiosyncratic risks. From the above test results we can draw several policy implications for further improvement of PPP system: rigorous ex-ante project assessment, inducement of market competition, proper private partner selection, and optimal risk sharing.

(1) Rigorous project assessment

PPP should be used to enlarge financing space for government, meaning it should be able to save tax payers money by securing value-for-money (Vfm). The Vfm can be obtained only when a life-cycle cost of PPP option is cheaper than a traditional public procurement, while maintaining same quality of service. To

⁴⁶⁾ the average bid-ratio of military group is 1.71, that of education is 2.10 and that of culture is 2.02

achieve this goal, quantitative assessment is needed to compare the government's burden in a professional and independent way. Korean government has been trying to control average BTL return within Nationally affordable level. From the legislation of BTL in 2005, ex-ante project assessment is mandated to secure value-for-money (Vfm). The 6% of discount rate is used in the Vfm test currently. The discount rate is one of the most important factors that determines Vfm. Other conditions being equal, if the discount rate increases, Vfm increases because the present value of government's future liability from BTL contract becomes lower. As the discount rate is higher than the optimal level, some of the projects lack of Vfm may have passed and exploit excess returns. Based on the study result, optimal level of average BTL return reflecting systematic risk is around 5.4% ~ 5.8%. As the current 6% of discount rate is higher than the optimal level, it had better be lowered. As a result, it may be suggested that a rigorous ax-ante project assessment with lower level of hurdle rate can contribute to affordable average of alpha in the National level, eventually resulting in public benefit. However, at the same time, bankable level of project return should be granted to invite numbers of willingness investors to PPP.

(2) Market competition and maturity

Promoting a market competition is one of the most important factors for the successful implementation of PPP. As was proved by the empirical analysis result, the bid ratio has significant impact on the level of risk spread (alpha). At the same time, it is revealed that the degree of market maturity (project experience) also affects on the level of risk spread. It is a good empirical evidence that the enhanced market maturity or competition can contribute to public good by decreasing required rate of return of private investors. By decreasing alpha spread, government's payment burden can be reduced. As a result, it is concluded that promoting voluntary decrease of alpha through market competition will be one of the most desirable goal for public policy.

Several policy means may be considered to increase the degree of market

competition. First of all, transparent public information release may contribute to the vitalized market competition. As was discussed, PPP information is not well published. There probably exists considerable information asymmetry between public and private, as well as among private investors, which fact defers market competition. Not a few cases, wilful investors may not be able to join a suitable project due to a lack of project information. Enhanced system of public hearings or public opinion collection process during project planning may be considered as one of examples for project information sharing.

Together with information transparency, simplifying and shortening the PPP procedure can be a second policy means for the market promotion. Typically, PPP projects need long preparation time, which can't be endured by middle or small cap investors. As is shown from empirical analysis result, there is no statistical evidence that project preparation period significantly influenced on the average contract alpha. It means the reduction of project preparation period through simplifying process and inviting efficiency can motivate broader range of private investment to join PPP without increasing average risk spread, alpha.

Additionally, there exist another indirect benefit from vitalized market competition. As is discussed in the empirical analysis, the correlation coefficient of debt funding cost spread (kd_spread) shows 0.187, which is one of the strongest exogenous variables. It is concluded that, if other conditions being equal and the debt funding cost spread (kd_spread) increases 1%p, the average alpha increases 0.187%p under 99% of confidence. The debt funding cost is determined by supply and demand of finance market. The fund demand is determined by accumulative value of PPP projects, while the fund supply is determined by number of willing financial institutions. If PPP market is vitalized and bigger number of financial investors look for investment opportunity, the supply curve of project financing (PF) market will shift to right, resulting in decreased funding cost in overall. The decreased funding cost can lower required BTL project return (alpha), resulting in fiscal budget savings.

(3) Proper partner selection and risk sharing

Another important policy implication may be selecting capable partner and optimally sharing risks to decrease total project risk. The lowered total risk level can directly impact on the required rate of return, and eventually on the government's budget. As is discussed, the optimal risk spread (alpha) level is determined mainly based on the amount of construction and operation risk. The detail meaning of 'risk' in this context is the 'forecasted' and at the same time, 'expected to be controlled'. Both procuring authority and private investor will forecast degree of risk when negotiating the level of alpha. During negotiation, the risk level is forecasted assuming it be allocated among procuring government and private equity-holders. Here arise two important issues: how much capable the project partners enough to manage allocated risks by providing experience and knowledge in their assigned roles, and how optimally the roles and risks are allocated to parties who can best handle. It means, even with same project, numerous cases of risk allocation can be made, and the total sum of allocated ex-post risk will be different depending on the quality of participants and risk sharing structures. This issue is closely related with the core rationale of PPP: even though the original project risk does not change, by properly allocating project risk to capable parties and by inviting private efficiency, the ex-post controlled risk can be reduced. On the basis of risk-return profile, if total risk reduced, required rate of return (or risk spread) shall decrease, too.

On the basis of empirical analysis, it is revealed that the stratified gaps of average contract alpha over environment sector are bigger than the differences of optimal levels among sectors. This result indicates the investors require comparatively higher excess returns in education and culture sectors than environment, implicitly recognizing additional idiosyncratic factors. But, if capable partners are selected and risks are allocated in better way, both construction and operation risks in ex-post basis may have decrease. And the lowered risk level can directly impact on the required risk spread of private investors. Recognizing the lower level of risk, private investors will probably agree decreasing project

alpha in negotiation. It may be achieved as number of projects accumulate and more experienced partners join PPP market. In summary, by selecting capable partners and sharing risk optimally, project return can be lowered without harming the possibility of funding, and eventually, government budget can be saved.

VII. Conclusion

'The Promotion of Private Capital into Social Overhead Capital Investment Act' was legislated in August 1994 as a way of supplementing government finances in Korea. Since then, the use of PPP has contributed to the rapid economic growth. As a result, Korea had total 435 BTL projects, which value amounted about \$24.77 billion as of 2013. The importance of BTL has been increasing due to the socio-economic trend. However, as the number of PPP projects accumulates, public criticism that private investors exploit too high profit from infrastructure is also increasing. Besides, most of researches commonly argue that infrastructure markets are inefficient and so, considerable level of excess returns exist for various reasons.

On the basis of the hypothesis test, it is found that the average contract risk spread (alpha) of BTL in Korea is higher than that of optimal level: the average contract alpha of environment sector is 0.17%p, military is 0.24%p, education is 0.26%p, and that of culture sector is 0.35%p higher than the optimal level of alpha. And it is concluded that there is no evidence BTL market of Korea is efficient⁴⁷⁾ in all sectors. To reduce the gap between the contract average and optimal level of alpha, more rigorous project assessment, using lower level of hurdle rate, is suggested as one of the policy implications. From the commencement of BTL in 2005, the value-for-money (Vfm) test has been performed, and currently 6% of discount rate is being used as a hurdle rate in the test. Based on the study result, optimal level of BTL return is estimated around 5.4% ~ 5.8%, considering systematic risk. As this rate is higher than the

47) However, it should be mentioned that the higher level of contract alpha than optimal level does not necessarily mean a mispricing because there exist considerable level of idiosyncratic factors that can not be removed through diversification in real world, especially in infrastructure market.

optimal level, some of the projects may have exploited excess return, although not substantial.

Empirical analysis is additionally performed to understand what idiosyncratic factors influence how much impact on BTL risk spread, using OLS (Ordinary Least Squares). The result provides supporting evidence of the different levels of idiosyncratic factors among facility sectors. The funding cost spread is proved to have the strongest impact on the average contract alpha. The lowest average funding cost of environment sector contributed to forming comparatively low contract alpha, while that of culture sector is high due to higher funding cost. It is also found that the average contract alpha has negative correlation with bid ratio. The average contract alpha of environment sector is supposed to be formed in lower level than other sectors due to the highest average bid ratio, which is 2.270⁴⁸⁾. It is observed the culture sector has the lowest maturity than other groups⁴⁹⁾, which fact may have contributed to the highest level of deviation of average contract alpha from the optimal level.

The empirical analysis reveals, regardless of the statistical confidence level, a project size does not have meaningful influence on the contract risk spread (alpha). This result suggests BTL investors do not require increased marginal return for bigger investment size. Or most of BTL projects are small sized and thus, the difference of investment sizes among projects are not significant. So, the marginal impact of investment size to alpha may have been limited. This finding also explains why bundling of BTL projects such as schools or dormitories are popular and efficient.

It is found that the stratified gaps of average alpha in, education, and culture sectors over environment based on empirical analysis are bigger than the gaps of

48) the average bid-ratio of military group is 1.71, that of education is 2.10 and that of culture is 2.02

49) Total population of culture group is only 42, which is far smaller than other groups: 96 in environment, 70 in military, and 218 in education.

optimal levels among sectors. Additional analysis is performed to compare the degree of alpha deviation from the regression line (model 3) in respective sectors. This result shows that investors in education and culture sectors requires higher excess returns implicitly recognizing additional idiosyncratic risks.

The risk free rate (r_f) and credit spread (aa_rf) variables are included in regression models for the control purpose. Anyway, from the coefficients of these two variables, it is found that the total return of BTL has some degree of rigidity against the change of both market rates. It implies that investors look for a long-term spread and thus, temporary decrease of market rate makes them require higher alpha in some degree to secure the total investment return over investable level, and vice versa when market rate increases.

Several PPP policies are suggested to achieve Nationally affordable level of BTL returns and thus, secure public benefit. As is discussed, more rigorous project assessment, using lower level of hurdle rate (lower than the current 6% of rate) is proposed as the first policy implication. As is proved from the analysis, the degree of market maturity or competition contributes to public good by decreasing required rate of return for private investors. By decreasing alpha spread, government's payment burden can be reduced. As a result, promoting voluntary decrease of alpha spread through market competition is another policy suggestion. The other policy implication is to select capable partner and optimally share project risk. The lowered risk level through proper risk sharing among capable partners can directly impact on the level of alpha, and eventually can save government's budget.

The research topic covered the optimal risk spread and idiosyncratic factors to draw policy implications. However, there remains further improvement points. This research measured operation risk depending on cross-sectional approach

because there is not enough time-series operation data of BTL projects. As time goes on, the financial data in operation phase will be accumulated and thus, the operation risk can be measured directly from respective SPC. Second, if data allows, it would be interesting to apply the framework of this paper to other economies (foreign markets) to compare the test results and degree of impacts by idiosyncratic factors.

REFERENCE

- Baccarini, D. & Archer, R., 2001, 'The Risk Ranking of Projects: a Methodology', International Journal of Project Management
- Beidleman, C., Flecher, D., Vesbosky, D., 1990, 'On Allocating Risk: The Essence of Project Finance', Sloan Management Review
- Bird, R., Liem, H. and Thorp, S., 'Infrastructure: Real assets and real returns', European Financial Management
- Brotherson, W., Keneth M., Robert S., and Robert, C., 2013, 'Best Practices in Estimating the Cost of Capital: An Update', Journal of Applied Finance
- Chen, Kensinger, Martin, 1989, 'Project financing as a means of preserving financial flexibility', University of Texas
- Choi, J., Park, T., 2013 'A study in the Reasonable Rate of Return for the Korean PPI Projects: An Investigation of Transportation Projects', The Seoul Institute (in Korean)
- Crosbie, P., Bohn, J., 2003, 'Modeling Default Risk', Moody's KMV
- Esty, B., 1999, 'A Case Study of the Effective use of Project Finance', Journal of Applied Corporate Finance
- Fama, Eugene F.; French, Kenneth R. (1992). 'The Cross-Section of Expected Stock Returns'. Journal of Finance
- Fama, Eugene F; French, Kenneth R (2004). 'The Capital Asset Pricing Model: Theory and Evidence'. Journal of Economic Perspectives
- Fama, Eugene F.; French, Kenneth R. (1993). 'Common Risk Factors in the Returns on Stocks and Bonds'. Journal of Financial Economics'
- Felsinger, K., Moranda, J., Silling, H., Booth K., 2009, 'Public-Private-Partnership Handbook', Asia Development Bank

- Finkenzeller, K Dechant T & Schafers W 2010, 'Infrastructure: a new dimension of real estate? An asset allocation analysis', Journal of Property Investment & Finance
- Fredric, B., & Roger S., 2007, 'How Banks Price Loans to Public-Private-Partnership: Evidence from European Markets', Journal of applied corporate finance
- Grimsey, D., & Lewis, K., 2000, 'Evaluating the Risks of Public Private Partnerships for Infrastructure Projects', International Journal of Project Management.
- Hamada, R., 1972, 'The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stocks', The Journal of Finance
- Hodge, A., 2004, 'The Risky Business of Public-Private-Partnerships', Australian Journal of Public Administration
- Jang, S., Kang, J., 2006, 'A Methodology for Determining a Reasonable Rate of Return on Private Investment in the Build-Transfer-Lease Scheme, Korea Transport Institute
- Joo J., et al., 2010, 'Analysis of Factors of IRRs and Spread on Korea's BTO Projects'
- KDI, 2012, 'Annual Report 2012' (in Korean)
- KDI, 2013, 'Annual Report 2013' (in Korean)
- Kim, J. 2013. 'Modularization of Korea, variety of activities into Partnerships: Lessons from Korea on Institutional Arrangements and Performance, KDI
- Lee, H., Park, S., 2014, 'Success Stories and Lessons Learned from Public-Private Partnership Projects in Korea', KDI
- Lev, B., 1974, "On the Association between Operating Leverage and Risk," Journal of Financial and Quantitative Analysis
- Ministry of Strategy and Finance, 2013, 'Comprehensive PPP evaluation 2012'

- Ministry of Strategy and Finance, 2015, 'Comprehensive PPP evaluation 2013'
- Ministry of Strategy and Finance, 2013, 'PPP Basic Plan 2013'
- Ministry of Strategy and Finance, 2014, 'PPP Basic Plan 2014'
- Meidute, I., & Paliulis, K., 2011, 'Feasibility Study of Public-Private-Partnership', International Journal of Strategic property Management.
- Park, J., Jeon, R., Kim, J., 2006, 'A Study on the Model for Determining the Optimal Risk Premium Considering Project Risk in BTL(Build-Transfer-Lease) Projects, Architectural Institute of Korea
- Regan, M., Smith, J. and Love, P., 2011 Infrastructure Procurement: Learning from Private-Public-Partnership Infrastructure projects, Journal of Construction Engineering and Management
- Robert C. Merton, 1972, 'On the Pricing of Corporate Debt: The Risk Structure of Interest Rates', Journal of Finance
- Robert, J., Michael, E., and Timothy W., 2014, 'The predictability of Australian Listed Infrastructure and Public-Private Partnership Returns: Using Asset Pricing Models.', CSIRO-Monash Superannuation Research Cluster
- Rothballer, C. and Kasrerer, C., 2012, The risk profile of infrastructure investments: Challenging conventional wisdom, Journal of Structured Finance
- Saunders, A., & Allen, L., 2010, 'Credit Risk Measurement In and Out of the Financial Crisis (third edition)', John Wiley & Sons, Inc.
- Sawant, R., 2010, The economics of large-scale FDI: The case of project finance, Journal of International Business Studies
- Sharpe, William F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk, Journal of Finance

- Shim, S., Choi J., Grimsey, D., Park, K., Choi, D., Lee, S., Hwang, S., 2006, 'Risk Management Measures of Private Investment in Infrastructure', KDI (in Korean)
- Shin, S., 2009, 'A Study on the Fair Returns of Private Participants' Investments', Korea Journal of Construction Engineering and Management
- Toms, S., 2012, 'Accounting based risk measurement: An alternative to CAPM derived discount factors'
- Vecchi V, et al. 'Does the private sector receive an excessive return from investments in health care infrastructure projects? Evidence from the UK', Health Policy (2013)
- Wang, Y., 2009, 'Structural Credit Risk Modeling', Society of Actuaries

APPENDIX

(1) Construction sponsor data statistics for beta calculation

Group	Number of project	Number of construction sponsors	Outlier	Number of sample	Selection ratio
Environment	96	126	14	112	88.89%
Military	70	79	4	75	94.94%
Education	218	185	7	178	96.22%
Culture	42	68	6	62	91.18%
Total	426	458	31	427	93.23%

(source : KIS line, Public and Private Investment Management Center)

(2) Equality test among estimated betas

Statistics

Statistics	average (in all BTL)	env	mil	edu	cult
$E(\beta)$	1.143	1.158	1.288	1.177	1.102
$SD(\beta)$	0.156	0.285	0.474	0.259	0.247
N	6,042	1,600	1,041	2,521	895

The hypothesis

$$H_o : \mu_1 = \mu_2$$

$$Z = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\delta_1^2}{N_1} + \frac{\delta_2^2}{N_2}}}$$

$$Z_{(\alpha=0.005)} = \pm 2.58, \quad Z_{(\alpha=0.025)} = \pm 1.96, \quad Z_{(\alpha=0.05)} = \pm 1.65$$

Z-test and conclusion

	$X_1 = \beta_a = 1.144$ $X_2 = \beta_{env} = 1.158$	$X_1 = \beta_a = 1.144$ $X_2 = \beta_{mil} = 1.288$	$X_1 = \beta_a = 1.144$ $X_2 = \beta_{edu} = 1.177$	$X_1 = \beta_a = 1.144$ $X_2 = \beta_{cult} = 1.102$
Z-value	-2.03	-9.78	-6.14	4.83
decision	$-2.58 < Z < -1.96$	$Z < -2.58$	$Z < -2.58$	$Z > 2.58$
c o n c l u s i o n	X_1 is different from X_2 (95% confidence)	X_1 is different from X_2 (99% confidence)	X_1 is different from X_2 (99% confidence)	X_1 is different from X_2 (99% confidence)
	H_0 is rejected under 95%	H_0 is rejected under 99%	H_0 is rejected under 99%	H_0 is rejected under 99%

(3) Calculation of risk free rate and long-term investment risk premium

	Rf_5years	Rf_20years	long-short spread (20-5)
2005-12-30	5.36%	N/A	N/A
2006-12-29	5.00%	5.19%	0.19%
2007-12-31	5.78%	5.67%	-0.11%
2008-12-31	3.77%	4.60%	0.83%
2009-12-31	4.92%	5.61%	0.69%
2010-12-31	4.08%	4.68%	0.60%
2011-12-30	3.46%	4.01%	0.55%
2012-12-31	2.97%	3.28%	0.31%
2013-12-31	3.23%	3.77%	0.54%
2014-12-31	2.28%	2.80%	0.52%
Average			0.457%

(4) Calculation of market return

There may be various ways of measuring market return depending on measurement period or asset coverage. This paper assumes the weighted average return on equity of all audited companies in Korea comprises local market return. Return on equity is selected because the estimated market

return will be used as a benchmark to BTL return on equity. And when market return is estimated from financial market, the return on stock (equity) is generally considered as a market return, which is consistent with accounting return on equity.

The net profit and equity amount data of all audited companies in Korea from 2000 to 2014 are collected. The return on equity of each company at time 't' are calculated by dividing profit with equity. All of the return on equity at time 't' are averaged by the weight of equity amount.

$$R(m)_t = \sum_{i=1}^{n_t} ROE_{(i)t} \times W_{(i)t}$$

$$\text{Where, } W_{(i)t} = \frac{\text{equity}_{(i)t}}{\sum_{i=1}^{n_t} \text{equity}_{(i)t}}$$

n_t = number of audited company at time 't'

$ROE_{(i)t}$ = return on equity of company (i), at time 't'

The result of market returns are summarized as below table:

Year	Number of companies (audited in Korea)	R(m) _t
FY 2000	11,255	3.86%
FY 2001	12,272	3.28%
FY 2002	13,325	8.84%
FY 2003	14,039	9.99%
FY 2004	14,597	14.38%
FY 2005	15,232	12.20%
FY 2006	16,172	10.30%
FY 2007	17,119	10.31%
FY 2008	17,985	4.72%
FY 2009	18,786	7.81%
FY 2010	19,877	10.07%
FY 2011	21,002	6.83%
FY 2012	22,124	5.94%
FY 2013	22,759	4.34%
FY 2014	21,262	5.19%
average		7.87%

(5) SPC selection data statistics for \triangle NOI estimation

Samples are selected when a project meets both criteria of operation status and showing positive NOI. Which means it is assumed that rational investors will forecast NOI based on operating projects that have normal, positive NOI. The sample selection ratio to available population is 78% in average (71% in environment, 92% in military, 75% in education, and 87% in culture facility)

	environment	military	education	culture	Total
Population (In operation & available data exist)	69	51	185	30	335
number of samples for regression	49	47	139	26	261
% of selected samples	71%	92%	75%	87%	78%

(6) Calculation of risk spread in operation (rs^*_{ot}) period using coupon spread ($_{0}cs_t$)

The risk spread calculated by time weighted average of coupon spread is compared with other result calculated using internal rate of return (IRR). In the below example, it is assumed that a \$500 of bond is comprised of 5 separate coupons with same amount (\$100) and scheduled to mature at every year. The cash flows from $t=0$ to $t=5$ are summarized in 'total CF' and the internal rate of return (IRR) is calculated.

In the right side, the time weighted average of coupon spread is calculated based on the below formula. The result is slightly different by 0.0002%, which seems reasonably waved. The risk spread in operation period is estimated by approximation with below formula.

$$rs^*_{ot} = \frac{\sum_{t=1}^T {}_0cs_t \times t}{\sum_{t=1}^T t}$$

(Example) comparison of results between time weighted average and IRR approach

	coupon rate	1st year	2nd year	3rd year	4th year	5th year	t * coupon rate
Coupon 1	0.800%	0.80					0.80%
Coupon 2	0.600%	0.60	0.60				1.20%
Coupon 3	0.050%	0.05	0.05	0.05			0.15%
Coupon 4	0.030%	0.03	0.03	0.03	0.03		0.12%
Coupon 5	0.020%	0.02	0.02	0.02	0.02	0.02	0.10%
Interest income		1.50	0.70	0.10	0.05	0.02	2.37%
Principle payment	500	100	100	100	100	100	$\sum_{t=1}^T t = 15$
Total CF	-500	101.5	100.7	100.1	100.05	100.02	
IRR =	0.1582%						$r = 2.37\% / 15$ $= \mathbf{0.1580\%}$

(7) Zero mean equality test on expected asset value volatility

- Statistics

Statistics	env	mil	edu	cult
$E(\frac{\Delta of Asset Value_j}{Asset value_i})$	0.0001958	-0.0006510	-0.0002595	0.0028926
$\delta(\frac{\Delta of Asset Value_j}{Asset value_i})$	0.00586	0.00762	0.01142	0.01411
N	49	47	139	26

- The hypothesis

$$H_o : \bar{X} \neq 0$$

$$t = \frac{(\bar{X} - 0)}{(\frac{\delta}{\sqrt{n-1}})}$$

- t-test and conclusion

	environment	military	education	culture
t-value (calculated)	0.23	-0.58	-0.27	1.03
d.f	48	46	138	25
t _{df} (99%)	±2.94	±2.95	±2.85	±3.08
conclusion	No evidence E(X) is different from zero (99% confidence)	No evidence E(X) is different from zero (99% confidence)	No evidence E(X) is different from zero (99% confidence)	No evidence E(X) is different from zero (99% confidence)
	H ₀ is rejected under 99%	H ₀ is rejected under 99%	H ₀ is rejected under 99%	H ₀ is rejected under 99%

(8) Θ ratio derivation

Financial theories generally state that next year's forward rate can be measured based on 2 and 1 year's spot rates, using geometric mean formula as below:

$$(1 + {}_0S_2)^2 = (1 + {}_0S_1) \cdot (1 + {}_1f_2)$$

Applying the same logic, the optimal life-cycle return of BTL can be expressed with a share of return for construction period, that for operation period, and a long-term risk spread as below:

$$(r_{*t}^* + 1)^m = (r_{c,t}^* + 1)^n \times (r_{o,t}^* + 1)^{m-n}$$

m : total project contract term

n : construction term

$m-n$: operation term

And the optimal return of BTL is defined as follows:

$$r_{*t}^* = \theta \cdot r_{c,t}^* + (1 - \theta) \cdot r_{o,t}^*$$

r_{*t}^* : optimal return from a BTL project assessed at time 't' (beginning of project)

$r_{c,t}^*$: optimal return during construction period from a BTL project

$r_{o,t}^*$: optimal rerun during operation period from a BTL project

Θ : share of return for construction period in total

If we get the solutions of all the exogenous variables except Θ , combining the 2 formula, we can express Θ as below:

$$[\theta \cdot r_{ct}^* + (1 - \theta) \cdot r_{ot}^* + 1]^m = (r_{ct}^* + 1)^n \times (r_{ot}^* + 1)^{m-n}$$

$$[\theta \cdot (r_{ct}^* - r_{ot}^*) + r_{ot}^* + 1]^m = (r_{ct}^* + 1)^n \times (r_{ot}^* + 1)^{m-n}$$

$$\theta \cdot (r_{ct}^* - r_{ot}^*) + r_{ot}^* + 1 = \sqrt[m]{(r_{ct}^* + 1)^n \times (r_{ot}^* + 1)^{m-n}}$$

$$\theta = \frac{[(r_{ct}^* + 1)^n \times (r_{ot}^* + 1)^{m-n}]^{\frac{1}{m}} - (r_{ot}^* + 1)}{(r_{ct}^* - r_{ot}^*)}$$

(9) Empirical test without risk free rate variable

VARIABLES	(1) alpha	(2) alpha	(3) alpha
aa_rf	-0.0523*** (0.0133)	-0.0532*** (0.0132)	-0.0516*** (0.0132)
kd_spread	0.173*** (0.0279)	0.173*** (0.0277)	0.199*** (0.0278)
invest_size	-0.0385** (0.0182)	-0.0357** (0.0177)	-0.0619*** (0.0154)
sub_binom	0.0268 (0.0414)		
bid_ratio	-0.0239** (0.0105)	-0.0225** (0.0104)	-0.0250** (0.0105)
authority	-0.0522 (0.0427)		
period	-5.97e-05 (6.72e-05)		
milit	0.0492 (0.0539)	0.0932*** (0.0348)	
edu	0.0601* (0.0306)	0.0763*** (0.0282)	
cult	0.193*** (0.0431)	0.192*** (0.0398)	
Constant	1.390*** (0.223)	1.284*** (0.206)	1.609*** (0.177)
Observations	329	329	329
R-squared	0.330	0.325	0.273

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

(10) The original expression of Merton model and application

The formula (4.11) in the paper is drawn based on the put option pricing formula expression of Black-Scholes because it is more familiar to practitioners. Even though the concept of option theory is similar, there is a technical difference in expressing cumulative

probability between Morton and Black-Scholes. The Black-Scholes does not consider the value of interest payment at maturity because the underlying asset is a stock, while the Merton model considers the payment of interest at maturity because the underlying asset is a bond.

The d_1 is expressed as $d_1 = \frac{\text{Ln}(V_t/L) + (r + \delta^2/2) T}{\delta \sqrt{T}}$ in Black-Scholes assuming $t=0$,

and if we consider additional interest payment at maturity, it will be changed as

$$d_1 = \frac{\text{Ln}(V_t/L) + (\delta^2/2) T}{\delta \sqrt{T}} .$$

Whereas, the original Merton model expresses $h_1 = - \frac{[\frac{1}{2} \delta^2 T - \ln(d)]}{\delta \sqrt{T}}$ (Where, $d = L/V_t$).

Which can be transformed as follows:

$$h_1 = \frac{-\frac{1}{2} \delta^2 T + \ln(d)}{\delta \sqrt{T}} = \frac{-\frac{1}{2} \delta^2 T - \ln(\frac{1}{d})}{\delta \sqrt{T}} = - \left[\frac{\ln(\frac{1}{d}) + \frac{1}{2} \delta^2 T}{\delta \sqrt{T}} \right] = - \left[\frac{\ln(V_t/L) + \frac{1}{2} \delta^2 T}{\delta \sqrt{T}} \right]$$

As a result, the h_1 in Merton model can be expressed as $(-d_1)$ when the term of Black-Scholes is applied as is in the formula (4.11).

Anyway, for the calculation purpose, the original form of Merton model is used in the paper. The original form of Merton model is expressed as below:

$$s = \left(\frac{-1}{\tau}\right) \ln [N(h_2) + \left(\frac{1}{d}\right) N(h_1)]$$

$$h_1 = - \frac{[\frac{1}{2} \delta^2 \tau - \ln(d)]}{\delta \sqrt{\tau}} \quad h_2 = - \frac{[\frac{1}{2} \delta^2 \tau + \ln(d)]}{\delta \sqrt{\tau}}$$

τ = the length of time remaining to loan maturity

d = the firm's (borrower's) leverage ratio measured as $\frac{Be^{-r\tau}}{A}$, where the market value

of debt is valued at rate 'r', the risk free rate of interest

$N(h)$ = a value computed from the standardized normal distribution statistical tables. This value reflects the probability that a deviation exceeding the calculated value of h will occur.