THE CAPITAL ASSET PRICING MODEL & A PORTFOLIO OF SMALL COMPANIES

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ABSTRACT

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Are risky stocks worth it? Do they offer better returns, in exchange for more risk? The modern portfolio theory holds that, yes, nondiversifiable risk (systemic risk) is rewarded by a higher expected return.

We collected weekly adjusted close ticker prices over seven years—from January 1999 to January 2006—for 29 US-listed small- or medium-capital companies, using the free data from Yahoo! Finance. Yahoo! Finance also provides a beta estimate for these companies' returns compared to the NASDAQ.

With this data set, CAPM-predicted returns can be compared with actual returns. It can be tested to see whether or not this portfolio's returns are to be expected. The NASDAQ Composite had a loss over the six years: it started at 2'344.41 in January 1999 and ended at 2'205.32 in January 2006. That's a drop of -0.059, or of about -5.9%.

We used a portfolio of 29 small-cap stocks as a proxy for the total small-cap population. In addition, we used Paulson's four companies and compared them to that larger small-cap population proxy. To get the weights of each stock, we looked at how much money Paulson invested in each one. The weights in the portfolio represented the amount of money Paulson put into each firm as the initial underwriter. The 10-year US Treasury note was our risk-free asset.

Our portfolio of companies offered above-predicted returns. Therefore, according to our data the CAPM does not hold: there are additional returns to be had. This may be because of Fischer's "small firm effect". This theory holds that smaller firms, or those companies with a small market capitalization, outperform larger companies. This market anomaly is a factor used to explain superior returns in the Three Factor Model created by Gene Fama and Kenneth French: the three factors are the market return, companies with high book-to-market values and small stock capitalization. In this case, the higher-than-expected returns seen by Paulson companies fit with the Three Factor Model.

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I would also like to thank Professor Lee Young-Ki and Professor Kim Pyung-Joo. Professor Lee's investment classes led to my current career in investment banking. He introduced me to the wonders of the market portfolio theory and the capital asset pricing model. Professor Kim's banking and monetary policy classes were wonderful; he makes an important subject exciting.

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INTRODUCTION

Long-term wealth is the goal of investing. Sometimes we want money a little quicker, but usually we can only make money slowly. Of course, investors want investments that bring wealth forever. That seems to be impossible. But some firms are more likely to bring wealth to their owners than others, at least for a short time. These are the “risky assets” in an investor’s portfolio. They are “risky” because their returns are not guaranteed and can seem to be unpredictable. US government debt is considered to be “risk-free”, since Washington, DC, has yet to default on a loan.

If we invest in a company—that is, if we become partial owners—how do we know if the market reflects its value. Are markets efficient? Specifically, are markets for risky assets efficient? What is a share worth? How much should I pay to own part of this firm? These are questions that run through the mind of every investor.

Paulson Investment Company is one such investor. It is a small-capital investment bank, similar to others. It specializes in high-risk, high-tech firms. It generally takes about six or seven companies public per year. It researches many more firms than it ends up not taking public. Paulson Investment works with firms that seek initial capital of normally less than USD 40 million.

Paulson, along with J.M. Dutton & Associates, sponsors a yearly conference in New York: the Westergaard SmallCap Conference. Year 2005 was the 28th Westergaard Conference. Thirty-one companies gave presentations, to seek finance. Paulson was the main underwriter for five of them. Institutional investors—fund managers, investment advisors, registered representatives, etc.—attend the Westergaard Conference to see presentations by these smaller firms. They look for investments that will offer above-market returns.

These firms are mostly traded on the NASDAQ exchange, a few in OTC markets. Their betas are calculated with regard to the NASDAQ Composite. Are these small firms worth the risk involved in investing in them?

BODY OF THESIS

Theoretical Review

Market prices offer the best way to determine the value of a firm or of a firm's assets, or property. This is important not only to those buying and selling businesses but also to regulators. An insurer, for example, may appear strong if it values the securities it owns at the prices it paid for them years ago, but the relevant question for judging its solvency is what prices those securities could be sold for if it needed cash to pay claims today.

In 1935, John Maynard Keynes is quoted as saying, "It is usually agreed that casinos should, in the public interest, be inaccessible and expensive. And perhaps the same is true of Stock Exchanges."[1] Some people may use stock markets as roulette wheels. But for all their shortcomings, stock markets remain the best way to bring people with money to invest together with people who can put that investment to productive use.

The shares of highly capitalized firms are traded frequently. Their prices often move from minute to minute. The path these movements follow is known to economists as a "random walk". This means that current or past share prices are

[1] http://www.tomments.com/

of no help in predicting future prices. The fact a share's price has risen (or fallen) does not mean that its next movement is likely to be up (or down).

Many price changes have no identifiable cause. They simply reflect the desires of two investors at a particular moment. But there are also price changes that can be attributed to the arrival of new information in the market. For example, a press release announcing that an aircraft manufacturer won a big order will boost its shares. But the higher price may not last as investors examine the customer's finances and conclude that it may not be able to afford the planes. The efficient market hypothesis contends that investors cannot make money trading on news reports and other public information, because the information is reflected in share prices as soon as it is known.

A stronger form of the efficient market hypothesis holds that share prices already incorporate all relevant information, whether public or non-public. If this were true, there would be no value in studying a company or an industry before deciding whether to buy shares. The evidence for this assertion, however, is weak. Although markets do act quickly on information, there are many anomalies, situations in which an astute investor is able to profit from identifying factors that are not yet reflected in a share's price.

William Shape published the capital asset pricing model (CAPM) in 1964. Parallel work was also performed by Treynor (in 1961) and Lintner (1965). The

CAPM extended Harry Markowitz’s portfolio theory to introduce the notions of systematic and specific risk. For his work on the CAPM, Sharpe shared the 1990 Nobel Prize in Economics with Harry Markowitz and Merton Miller.

The CAPM decomposes a portfolio’s risk into systematic and specific risk. Systematic risk is the risk of holding the market portfolio. As the market moves, each individual asset is more or less affected. To the extent that any asset participates in such general market moves, that asset entails systematic risk. Specific risk is the risk which is unique to an individual asset. It represents the component of an asset’s return which is uncorrelated with general market moves.

According to the CAPM, the marketplace compensates investors for taking systematic risk but not for taking specific risk. This is because specific risk can be diversified away. When an investor holds the market portfolio, each individual asset in that portfolio entails specific risk, but through diversification, the investor’s net exposure is just the systematic risk of the market portfolio.

Systematic risk can be measured using beta. According to the CAPM, the expected return of a stock equals the risk-free rate plus the portfolio’s beta multiplied by the expected excess return of the market portfolio. Stated another way, the stock’s excess expected return over the risk-free rate equals its beta times the market’s expected excess return over the risk free rate.

This relationship is the essential conclusion of the CAPM. It states that a stock's (or portfolio's) excess expected return depends on its beta and not on its volatility. Stated another way, excess return depends upon systematic risk and not on total risk.

We call the CAPM a "capital asset pricing model" because, given a beta and an expected return for an asset, investors will bid its current price up or down, adjusting that expected return so that it satisfies the formula. Accordingly, the CAPM predicts the equilibrium price of an asset. This works because the model assumes that all investors agree on the beta and expected return of any asset. In practice, this assumption is unreasonable, so the CAPM is largely of theoretical value. It is the most famous example of an equilibrium pricing model.

Beta is a measure of a share's price volatility, relative to the average volatility of the national stock market. A share with a beta of 1.0 will, on average, move in tandem with the market average; a share with a beta of 1.5 can be expected to rise (or fall) 1.5% when the market rises (or falls) 1%. A share with a negative beta moves, on average, in the opposite direction from the market.

A high positive beta signifies a risky share that can be expected to outperform the market in good times but fall more than the market in bad times. The shares of many small firms, so-called small-cap stocks, carry high betas. A stock with a positive beta of less than 1.0 is a conservative investment; it is safer in a falling market, but offers less potential for appreciation when the market is rising. Shares with negative beta are for contrarians who want stocks that are likely to rise as the market falls. The betas of widely traded shares can be found in many investment periodicals and in research reports issued by stockbrokerage firms.

Pricing an initial public offering (I.P.O.) can be difficult. For example, in April 2006 Paulson brought public American Mould Guard (NASDAQ:AMGIU). The company's shares started trading at USD 13.50. How do companies determine this price?

Many people, in particular Fama and French in 1996[2], have examined the difficulties in pricing I.P.O.s. Generally, it seems as if I.P.O.s outperform in the short run, but underperform in the longer run. Part of the difficulty in pricing I.P.O.s is the information asymmetry. Venture capital firms, like Paulson, specialize in collecting and evaluating information on start-up, or "growth", firms. Alon Brav and Paul Gompers have done a study[3] that looked at whether there are longer-term effects associated with the presence of venture capitalists when a company goes public. In favour of venture capitalists/ underwriters, the authors say that venture capitalists: stay on the board of directors long after the I.P.O.; enforce management structures that help the firm in the long run; affect who

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[2] Eugene Fama and Kennth French, "Multifactor Explanations of Asset Pricing Anomalies," Journal of Finance, 1996.
[3] Alon Brav and Paul Gompers, "Myth Or Reality? The Long -Run Underperformance of Initial Public Offerings: Evidence for Venture and Nonventure Capital-Backed Companies", Journal of Finance December 1997.

hold’s a company’s shares; and improve the “reputation”, or “credibility”, of the new company.

As mentioned in the introduction, the successful returns seen by Paulson-invested companies is in line with Fama & French’s Three Factor Model. This model holds that smaller companies have greater growth opportunities than larger companies. These smaller firms also have more volatile business environments; they may be in high-tech or in other emerging industries. By correcting these problems, i.e., receiving a large amount of new capital, these companies can see a large, short-term appreciation: their stock price will go up more than expected if they get a large amount of new capital.

As mentioned above, Paulson recently brought another company public with a stock price of USD 13.50. This low price normal for small-capital companies. They tend to have lower stock prices. Lower stock prices allow for more potential for upward movement, and higher percentage rates of growth if the company does well.

The CAPM

The CAPM lets us test whether or not a company’s risk is rewarded by more successful returns. There a few assumptions required to test the capital asset pricing model (CAPM). Any finance textbook would list them in its first heading under "CAPM". These assumptions create an ideal world. This is good for testing economic theories, less good for real-world investing.

The usefulness of the CAPM is still not decided. Some studies support the CAPM: Black; Black, Jensen & Scholes; and Fama & MacBeth. Some studies challenge the CAPM: Banz; and Fama & French. Then, some studies even challenge those challenges: Amihud, Christensen & Mendelson; Black; Breen & Korajczyk; Jagannatham & Wang; Kothari, Shanken & Sloan. Even though the academic debate continues, we believe the CAPM may still be useful for long-term value investors.

To derive the CAPM, we start by choosing an arbitrary portfolio of assets. We let R_f be the return on the risk-free asset. By investing USD 1, the investor will be guaranteed a return of $\$R_f$. Beyond the risk-free asset, we use a formula to calculate the expected returns. K_c is the risk-adjusted discount rate (also known as the Cost of Capital). K_m is the return rate of a market benchmark, such as the NASDAQ Composite.

If we're using a sample portfolio, the CAPM theory works out to this:

$$K_C = R_F + \beta[K_M - R_F]$$

The K_c is the estimated return of our sample portfolio. Using the weights of Paulson investments in each firm, we will use Paulson-invested companies as our sample portfolio. Paulson only invested in four of the 29 companies.

For an individual asset, the CAPM uses slightly different notation:

$$\bar{r}_a = r_f + \beta_a(\bar{r}_m - r_f)$$

The r_a is the expected return for a given asset, in our case a stock price. The β_a is the individual stock's beta compared to its market, normally the NASDAQ.

The two formulas are slightly different, one for a portfolio and one for an asset. The common element in both equations is the risk free rate. For us, that's a 10-year US Treasury bond. The two formulas explain a similar relation between assets or portfolios and a larger market. We can turn this formula around in our heads to understand what it means for investors: they need to be compensated for both the time value of their money and for the risk they take on.

The time value of money shows up with the risk-free rate (r_f). This is what the investor could get otherwise if he put his money in a 10-year US Treasury bond.

The other half of the formula ($\beta_a(\bar{r}_m - r_f)$) shows risk. It calculates the amount of compensation the investor needs to take on for any risk above and beyond that of the general market. It's a variation of a discounted cash flow. According to our data, 10-year US Treasury are offering rates of 4.95% but the NASDAQ only grew by 0%[4].

[4] As found at Yahoo! Finance: http://finance.yahoo.com/bonds

Each company has a different beta, as calculated by Capital IQ, a company that supplies Yahoo! Finance with financial data. The security market line (SML) must show the risk premium for each stock. It's a relation between a company's beta and the expected return. (Please see Table 6, on page 24.)

To test the CAPM and the portfolio of small-cap firms, we will look at our portfolio of 29 Westergaard companies. In this portfolio, each company is given an equal weight. This portfolio of Westergaard companies can be thought of as an index of small-cap companies to be compared to the larger NASDAQ. The average beta for all 29 firms was 2.072. (Please see Appendix 2, page 37.)

$$K_C = R_F + \beta[K_M - R_F]$$

$$K_C = 4.95 + \beta[-0.06 - 4.95]$$

$$= -5.431$$

The top five actual change in stock prices between January 1999 and January 2006 were seen by the following firms: TASR (1'251.84%, from \$0.53 to \$6.96), BIPH.OB (375.00%), DSTI (289.58%), INMD (243.90%) and ISR (206.63%).

The Portfolio

Individual investors have control over only one aspect of their investment performance: the allocation of assets among broad asset categories. Nobody controls the stock market, and no one has direct control over the value of their shares or bonds. Small-cap stocks are only one type of asset category.

An investor must first make the capital allocation decision. This is the balance between risk-free assets (such as bonds, which are, theoretically, risk-free) and risky assets (such as our stocks). Riskiness is more interesting, particularly from a researcher's point of view; the dream of wealth is large.

Most investors will have at least some of their money in risky assets. Normally, these risky assets are publicly traded shares. This is for investors who already have a comfortable amount of savings in risk-free/ safe assets. This portfolio is for those investors with a certain degree of risk tolerance. The portfolio made in step one is a possible proxy for all small-cap firms in the US, compared to the NASDAQ. Our expected return is -5.431. Why is this so?

If we buy only one stock, we are exposed to that one company's firm-specific risk. This is diversifiable, however. So we design a portfolio of risky assets. However, this total risk is only diversifiable to a point. We cannot diversify away all the systemic risk.

Similarly, if we buy only one stock, we are exposed to that one firm's reaction, or response, to the market. If an estimated market goes up or down, the company's share price will generally go up or down as well. This is called systemic risk, measured by “beta”. We can also design our portfolio of risky assets to take into account each firm’s different reaction to the market. With one security, we are exposed to both undiversifiable systemic risk and diversifiable or firm-specific risk. However, by constructing a portfolio with multiple securities, we can reduce the total risk by diversifying away the non-systemic risk. Our theoretical portfolio of risky assets was made up of four companies that Paulson brought public or helped to bring public in 2005.

Once we have a portfolio—even an optimal portfolio—how do we know if it did any better than the market as a whole, or than any other portfolio? To determine that, in the final part of this thesis, we test whether these stocks actually performed better than either the S&P 500 index or the NASDAQ Composite index. Our portfolio produced a CAL inferior to the CML. This larger portfolio of small-cap, risky stocks was unsuccessful—if it were a portfolio—compared to the NASDAQ.

With the total stock price change from January 1999 to January 2006 for each of the 29 firms that presented at the Westergaard Conference, we can compute the mean return and use this sample mean to estimate the mean return over time to each firm. In other words, we can estimate a population parameter from our single sample.

We are trying to show whether the 29 stocks have generated returns commensurate with their systemic risk (beta). Though Paulson itself only directly financed 5 of these firms, we are using the larger portfolio of 29 stocks which Paulson worked with at the recent Westergaard small-cap conference.

Our population is all small- or mid-cap stocks listed in the US, particularly small-cap risky firms in emerging technologies. Our parameter—that which matters to investments—is the value of the share price.

A confidence interval is a range of values within which we expect the population parameter to occur. Our portfolio was tested at the 90%, 95% and 99% confidence levels.

In terms of investment advisory services, this is what some research analysts do for their clients: design portfolios. Asset managers use CAPM, betas and market to tailor portfolios to their clients' needs and risk-tolerance levels. After the hypothesis test, the final step in this process, an investment advisor would not be able to say, "I'm 90% certain that a mixed portfolio of these 29 companies would have given you a return of 0.17% between 1999 and 2006."

US finance industry regulation is quite strong. As an investment advisor I'm legally allowed to make the above statement only if, basically, I did the research myself. The math and number-crunching is proprietary and the investment advisor must tailor each statement to each client's needs, tolerance for risk and net worth. The two large self-regulatory organizations (SROs) in the US, the NASD and the SEC, are quite strict.

Equities can offer better returns than more staid investments, like US Treasury bonds. The NASDAQ Composite during our time period went down by about five percent. However, the share prices of many of our individual firms saw high growth, as reflected in Yahoo! Finance. Prices were adjusted the stock prices for dividends or splits.

The capital asset pricing model (CAPM) is our most common asset-pricing model: it tells us what our investments are worth. Construction firms use it for project management and valuation; investment advisors use it to value their portfolios. It's just a way to value things. It determines a "hurdle rate", the required rate of return for a project, through a discounted cash flow analysis. An investment makes sense—and money—above this rate. For our clients, the required rate of return is often the market's rate of return; any lower, and the stock broker is fired.

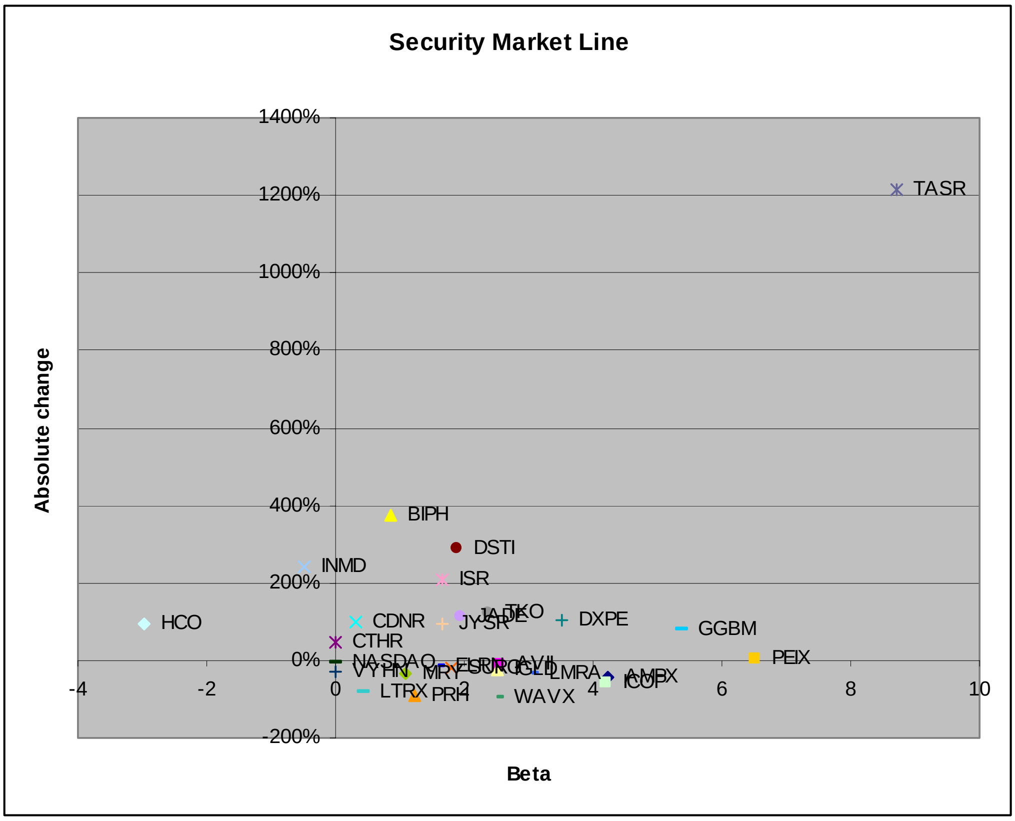
So our CAPM gives us a theoretical relationship between the risk of our portfolio and its expected return. It helps answer the question, "Is this worth it?" This gives us a benchmark target, allowing us to value as-yet undeveloped resources (i.e., portfolios we're going to design in the future).

We need to know how and why it fails. If the CAPM fails, it does not necessarily mean that the market is efficient, according to Rolls[5]. The CAPM may appear to be rejected, not because it is wrong, but because the NASDAQ Composite—or whatever one uses as a "market"—is not close enough to the true market portfolio available to investors. For example, investors can always put their money in gold or real estate, two factors not accounted for in our CAPM "market". The NASDAQ, or any index, is only a sample of the greater market.

The expected return of a project with a beta (β) of zero is simply the risk free rate (R_F). Six of our companies had a beta between -0.5 and +0.5. The expected return of a project with a beta (β) of 1 is the same as the expected return on the market (K_m). Only two of our companies had a beta between +0.5 and +1.5.

[5] Roll, Richard. . "A Critique of the Asset Pricing Theory's Tests: Part I: On Past and Potential Testability of the Theory." *Journal of Financial Economics.* 4 (March 1977): 129-76.

Table 1—Beta versus Absolute stock price change



The Hypothesis Test

Paulson had a theoretical portfolio of USD 71.125 million, as per Table 4. Of Paulson companies that presented at the Westergaard Conference, only four of them can constitute a theoretical Paulson investment portfolio. In all three cases, the actual returns were higher than the CAPM-predicted returns.

Table 2—Paulson Theoretical Portfolio

Company	IPO investment (USD, million)	Betas	Difference (between actual returns and CAPM-predicted returns)
ICOP	10.725	4.21	-39.67
I-Sector	8.3	1.65	209.95
Lumera	41.7	3.05	-24.63
Taser	10.4	8.70	1251.84

Paulson's total investment in these four firms was USD 71.125 million. This was Paulson's portfolio of underwritings, though the firm held other assets in addition to these.

The population of the stock market is too large to feasibly study every single potential company in a total population. There are too many publicly listed companies in the world. The results would not be worth the effort to gather data on every company in the world, every company in the US, every small-cap company in the US or even every small-cap risky company listed in the US[6].

Reuters has a few built in company lists at its finance news site. There is a "US Companies—All Small Cap" pre-fabricated list. Reuters shows 779 companies in this list. That is a large number of companies.

[6] Reuters Finance, for example, has a feature at its homepage called the "PowerScreener". From the Reuters Investing homepage (http://today.reuters.com/news/home.aspx), we can click on "Ideas & Screening" in the left margin. This brings us to the "Ideas & Screening" page (http://www.investor.reuters.com/ArticleEntry.aspx?target=%2fopinion).
Again along the left, click on "PowerScreener Lite". This is Reuters' free version of its PowerScreener. You need to create a free username/ password combination to use this feature.

An alternative to compiling and researching each of Reuters' 779 small-cap companies is to use a sample. To narrow this, we simply chose firms that were involved with Paulson Investment when they went public.

We need to keep in mind that our sample may be biased. Perhaps Paulson only selects particularly successful companies to work with, or particularly bad companies. Paulson is one of the few, independent underwriters. It was founded in the early 1970s and has a record of successful underwritings. So perhaps a list of companies underwritten by Paulson would show a bias toward successful firms.

We can test a statement to determine whether the sample does or does not support the statement that small-capital stocks offer a better return than the S&P average.

The portfolio beta is the weight of IPO investment Paulson invested into each firm. ICOP had 15.08% of this total portfolio, I-Sector 11.67%, Lumera 58.63% and Taser 14.62%. The beta for this portfolio would be 3.89.

Our first step is to state the null and alternate hypotheses. We believe Paulson's small portfolio of 4 underwritings will outperform either the NASDAQ Composite index.

$$H_0 : Exp.ret._{PAUL} \leq Exp.ret._{NSDQ}$$

$$H_1 : Exp.ret._{PAUL} > Exp.ret._{NSDQ}$$

For our test statistic, we cannot use *z*. The standard normal distribution can only be used if our population is known to follow a normal distribution and if we know the real, complete standard deviation for the whole population. As mentioned above, there are too many listed companies in the world to get "real" population data. Second, we have much fewer than 30 samples. Finally, we don't know our standard deviation. So we replace the standard normal distribution with what's called a *t* distribution. It's a little flatter and more spread out (wider) than the *z* distribution. As our sample increases—that is, as our degrees of freedom increase—the *t* distribution begins to look more and more like the *z* distribution.

The *t* formula is below.

$$t = \frac{\overline{X} - \mu}{s / \sqrt{n}}$$

The μ is our hypothesized population mean. This was the mean for the hypothesized population of 29 companies. The average return for those companies was 16.98%. The n is the number of observations in our sample, 4. To get our critical value, our degrees of freedom would be 3, or $n - 1$.

The mean of our sample is $\overline{X}$. For us, this is the mean return for our four Paulson-invested companies, 2'563.72%. Please see below. For example, I-Cop's stock price between January 1999 and January 2006 went from $15.5 to $6.85, a drop of 55.81%. The other numbers are below.

Table 3—Paulson Theoretical Portfolio stock price change

Company	Stock price in Jan. 1999	Stock price in Jan. 2006	% Δ
ICOP	15.5	6.85	-55.806
I-Sector	1.81	5.55	+206.630
Lumera	0.53	6.96	+2’477.850
Taser	5.75	3.74	-64.957

The s is the standard deviation of our sample. The standard deviation of our portfolio, weighted by the amount of money Paulson put into each company, was 112.92%. Please see the calculations and table below.

$$= \frac{(15.075 * 0.832) + (11.67 * 11.632) + (58.63 * 2.562) + (14.62 * 10.477)}{4}$$

$$= \frac{12.54240 + 135.74544 + 150.21006 + 153.17374}{4}$$

$$= \frac{451.67164}{4}$$

$$= 112.91791$$

Table 4—Paulson Theoretical Portfolio standard deviation of stock price

Company	Average weekly change %	Standard deviation %
ICOP	0.588%	0.832%
I-Sector	-2.707%	11.632%
Lumera	2.681%	2.562%
Taser	1.956%	10.477%

For our hypothesis test, we will have two levels of significance: 95% and 99%. The 0.05 (or 95%) level is used for consumer research projects. The 0.01 (99%) level for quality assurance. The more lax 0.10 (90%) level is used for political polling. We want to make sure our portfolio will, actually, out-perform the market, so we will stick with 95% and 99%.

The decision rule states the conditions under which H_0 is rejected. This requires calculating the critical value. It's simply a number that acts as a dividing point between two areas under the curve. A decision is made whether to reject or not reject the null hypothesis based on this critical value.

Our *p*-value expresses the likelihood that our H_0 is not true. It's the chances of seeing a sample value as extreme as, or more extreme than, that which we observed, given that the null hypothesis is true.

The critical values of *t* are given in any *t* distribution chart. A portion of the *t* distribution chart is below.

Table 5—Critical Values for Hypothesis Test

df	Confidence Intervals			
	90%	**95%**	**98.5%**	**99%**
	Level of Significance for One-Tailed Test			
	0.01	**0.05**	**0.020**	**0.01**
3	1.638	2.353	3.182	4.54070

Our test is one-tailed. We are testing to see whether our optimal risky portfolio has *greater* returns than the market. We are not testing whether our optimal risky portfolio merely has a result *different from* the market.

For our 99% confidence level and our 3 degrees of freedom, the value is 4.541. Because this is a one-tailed test and the rejection region is to the left of our distribution, the critical value must be negative. The decision rule is to reject H_0 if the value of *t* is less than that.

To see if our optimal, restricted (no short sales or margin accounts) risky portfolio return is likely to happen, we use the following data: $\overline{X} = 2'563.72\%$, the sample mean; $\mu = 16.98\%$, the hypothesized population mean; $s = 112.92\%$, the sample standard deviation; and $n = 4$, the number of observations.

The value of *t* is quite large, at -20.40. It is found by the formula below:

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{2'563.72 - 16.98}{112.92/\sqrt{4}} = \frac{2'546.74}{112.92/2} = \frac{2'546.74}{56.46} = 45.11$$

Because 45.11 lies in the region to the right of the critical value, the null hypothesis is accepted at either significance level. There is no statistically significant difference between $\overline{X}$ and μ. This indicates that Paulson's portfolio will not necessarily give better results than the greater population sample of Westergaard companies.

Data used.

There were 30 companies that presented at the Westergaard conference in 2005. One of these, Chinamerica Fund, is a portfolio of three other companies, making the company total 32. Three had no publicly available stock price data at Yahoo! Finance. Those three are marked with an asterisk in the table below. Our total number of company data lists, then, is 29.

Paulson was the main underwriter for five of these: ICOP, I-Sector, Lumera, NuVim and Taser. Below is a list of the investment Paulson made in each of the five.

Table 6—Recent Paulson Investments

Company	Initial investment (USD, million)
ICOP	10.725
I-Sector	8.3
Lumera	41.7
NuVim	2.7
Taser	10.4

Not all the companies at the Westergaard Conference have complete stock price history from January 1999 until January 2006. BAK Li-Ion Battery (over the counter bulletin board (OTCBB) ticker symbol: CBBT), for example, has barely one year of stock price data. Only eight of the companies have complete data for all seven years (all of 1999, 2000, 2001, 2002, 2003, 2004 and 2005).

SUMMARY & CONCLUSIONS

Based on the above, we should feel wary of small, risky companies. The portfolio of Paulson companies showed greater-than-market returns. But, as per the hypothesis test, this may not be true every time. Our results were based on past data and similar returns may not happen in the future. Data from Fama and French show that small-cap value stocks have outperformed the broader market by more than five percentage points since 1927[7]. It seems our data fits with their results. But, we still failed the hypothesis test: are results may be an anomaly.

As we said in the introduction, wealth for the long-haul is the goal for most investors; getting rich slowly is the surest way to do so. But some firms are more likely to bring wealth than others, at least for a short time. These are the "risky assets" in any investor's portfolio. Our portfolio of Paulson-invested companies has been shown to bring wealth, at least for the seven years from 1999-2006. Our sample population, of Westergaard companies, grew, too. We cannot reject the null, even though our portfolio's and population's expected returns are greater than the market's.

There are a few actions we could take to make our research more thorough.

[7] Fama and French, 1992, The cross-section of expected stock returns, Journal of Finance 47, 427-465.

- We could follow our 29 stocks throughout 2005 and into 2006. More data would allow us to see whether or not any one stock's returns were abnormal.
- Paulson Investment Company has also recently found two more firms which could be used as parts of its investment portfolio. These could be added, using data as far back as January 1999.
- Our sample size of 4 is very small. A better hypothesis test could be done if we looked at all Paulson-underwritten companies since the company opened, in 1971. This larger group, compared to Reuter's list of small-cap companies, for example, would give us a stronger answer to our hypothesis test.
- Our data during this time frame is unique. Our risk-free asset gave better returns than the market: ten-year US Treasury notes currently have a 4.95% yield; both the S&P 500 and the NASDAQ saw only 0% growth over the past seven years.

APPENDIXES

Appendix 1

Twenty-nine small- and mid-cap promising investment opportunities

Company	***Market: Ticker***	Homepage
Ampex Corp.	Nasdaq: AMPX	www.ampex.com
AVI BioPharma Inc.	Nasdaq: AVII	www.avibio.com
Biophan Technologies, Inc.	OTCBB: BIPH.OB	www.biophan.com
Cadence Resources Corp.	OTCBB: CDNR.OB	TBA
Charles & Colvard Ltd.	Nasdaq: CTHR	www.moissanite.com
Chemokine Therapeutics Corp.	OTCBB: CHKT.OB*	www.chemokine.net
Chinamerica Fund, LP		www.chinamericafund.com
China Digital Wireless	OTCBB: CHDW.OB	
BAK Li-Ion Battery	OTCBB: CBBT.OB	
China Agritech, Inc.	OTCBB: CAGC.OB*	
DayStar Technologies, Inc.	Nasdaq: DSTI	www.daystartech.com
DXP Enterprises Inc.	Nasdaq: DXPE	www.dxpe.com
Elron Electronic Industries Ltd.	Nasdaq: ELRN	www.elron.com
GigaBeam Corp.	OTCBB: GGBM.OB	www.gigabeam.com
HyperSpace Communications Inc.	AMEX: HCO	www.ehyperspace.com
ICOP Digital Inc.	Nasdaq: ICOP	www.icopdigital.com
IntegraMed America Inc.	Nasdaq: INMD	www.integramed.com
Internet Gold Golden Lines Ltd.	Nasdaq: IGLD	www.igld.com
I-Sector Corp.	AMEX: ISR	www.i-sector.com
Joystar Inc.	OTCBB: JYSR.OB	www.joystar.com
Lantronix, Inc.	Nasdaq: LTRX	www.lantronix.com
LJ International Inc.	Nasdaq: JADE	www.ljintl.com
Lumera Corp.	Nasdaq: LMRA	www.lumera.com
Memry Corp.	AMEX: MRY	www.memry.com
NuVim Inc.	OTCBB: NUVM.OB*	www.nuvim.com
Pacific Ethanol, Inc.	Nasdaq: PEIX	www.pacificethanol.net
Synergetics USA, Inc.	Nasdaq: SURG	www.synergeticsusa.com
TASER International Inc.	Nasdaq: TASR	www.taser.com
Telkonet, Inc.	AMEX: TKO	www.telkonet.com
Vita Cube Systems Holdings, Inc./ XELR8 Holdings, Inc.	AMEX: PRH	www.xelr8.com
Vyteris Holdings Inc.	OTCBB: VYHN.OB	www.vyteris.com
Wave Systems Corp.	Nasdaq: WAVX	www.wave.com
XAAR Plc.	LSE: XAR	www.xaar.co.uk

Appendix 2

CAPM Test For Individual Companies

Company	Market: Ticker	Beta	Risk-free rate of interest	Expected return of the market	Expected return (by CAPM)	Average standard deviation (%)	Actual change, from Jan. 1999 to Jan. 2006(%)	Difference (between actual return and expected return)
			4.95	**-0.06**				
Ampex Corp.	Nasdaq: AMPX	4.22	4.95	-0.06	-16.19	11.83	-45.63%	-29.44
AVI BioPharma Inc.	Nasdaq: AVII	2.55	4.95	-0.06	-7.82	-1.94	-12.44%	-4.61
Biophan Technologies, Inc.	OTCBB: BIPH.OB	0.87	4.95	-0.06	0.59	15.08	375.00%	374.41
Cadence Resources Corp.	OTCBB: CDNR.OB	0.31	4.95	-0.06	3.40	8.58	100.88%	97.49
Charles & Colvard Ltd.	Nasdaq: CTHR	0.00	4.95	-0.06	4.95	7.18	44.54%	39.59
Chemokine Therapeutics Corp.	OTCBB: CHKT.OB*	n/a	4.95	-0.06	n/a	n/a	n/a	n/a
Chinamerica Fund, LP		n/a	4.95	-0.06	n/a	n/a	n/a	n/a
China Digital Wireless	OTCBB: CHDW.OB	n/a	4.95	-0.06	n/a	20.93	-61.45%	n/a
BAK Li-Ion Battery	OTCBB: CBBT.OB	n/a	4.95	-0.06	n/a	n/a	n/a	n/a
China Agritech, Inc.	OTCBB: CAGC.OB*	n/a	4.95	-0.06	n/a	n/a	n/a	n/a
DayStar Technologies, Inc.	Nasdaq: DSTI	1.88	4.95	-0.06	-4.47	14.83	289.58%	294.05
DXP Enterprises Inc.	Nasdaq: DXPE	3.50	4.95	-0.06	-12.58	9.68	105.25%	117.83
Elron Electronic Industries Ltd.	Nasdaq: ELRN	1.60	4.95	-0.06	-3.06	4.19	-12.79%	-9.72
GigaBeam Corp.	NasdaqSC: GGBM	5.38	4.95	-0.06	-22.00	4.04	78.82%	100.82

HyperSpace Communications Inc.	AMEX: HCO	- 2.98	4.95	-0.06	19.88	10.04	94.72%	74.84
ICOP Digital Inc.	Nasdaq: ICOP	4.21	4.95	-0.06	-16.14	0.83	-55.81%	-39.67
IntegraMed America Inc.	Nasdaq: INMD	- 0.48	4.95	-0.06	7.35	11.09	243.90%	236.54
Internet Gold Golden Lines Ltd.	Nasdaq: IGLD	2.50	4.95	-0.06	-7.57	8.82	-26.18%	-18.61
I-Sector Corp.	AMEX: ISR	1.65	4.95	-0.06	-3.32	11.63	206.63%	209.95
Joystar Inc.	OTCBB: JYSR.OB	1.66	4.95	-0.06	-3.37	6.94	94.12%	97.48
Lantronix, Inc.	Nasdaq: LTRX	0.44	4.95	-0.06	2.75	14.09	-79.38%	-82.12
LJ International Inc.	Nasdaq: JADE	1.93	4.95	-0.06	-4.72	8.36	115.72%	120.44
Lumera Corp.	Nasdaq: LMRA	3.05	4.95	-0.06	-10.33	2.56	-34.96%	-24.63
Memry Corp.	AMEX: MRY	1.09	4.95	-0.06	-0.51	8.07	-32.03%	-31.52
NuVim Inc.	OTCBB: NUVM.OB*	0.00	4.95	-0.06	4.95			-4.95
Pacific Ethanol, Inc.	Nasdaq: PEIX	6.17	4.95	-0.06	-25.96		3.54%	29.50
Synergetics USA, Inc.	Nasdaq: SURG	1.60	4.95	-0.06	-3.06		-21.22%	-18.15
TASER International Inc.	Nasdaq: TASR	8.70	4.95	-0.06	-38.63	10.48	1213.21%	1251.84
Telkonet, Inc.	AMEX: TKO	2.36	4.95	-0.06	-6.87	3.34	123.12%	129.99
Vita Cube Systems Holdings, Inc./ XELR8 Holdings	AMEX: PRH	1.22	4.95	-0.06	-1.16	3.65	-90.78%	-89.62
Vyteris Holdings Inc.	OTCBB: VYHN.OB	0.00	4.95	-0.06	4.95		-28.13%	-33.08
Wave Systems Corp.	Nasdaq: WAVX	2.51	4.95	-0.06	-7.62	9.77	-97.16%	-89.54
XAAR Plc.	LSE: XAR	n/a	4.95	-0.06	n/a	7.63	219.07%	n/a

n/a = data not available

Appendix 3

Data Analysis—Regression

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.42
R Square	0.18
Adjusted R Square	0.14
Standard Error	2.35
Observations	27.00

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1.00	29.21	29.21	5.31	0.03
Residual	25.00	137.57	5.50		
Total	26.00	166.79			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.01	0.61	-0.01	0.99	-1.26	1.25	-1.26	1.25
X Variable 1	0.45	0.20	2.30	0.03	0.05	0.86	0.05	0.86

RESIDUAL OUTPUT

Observation	*Predicted Y*	*Residuals*	*Standard Residuals*
AMPX	1.91	-2.36	-1.03
AVII	1.15	-1.28	-0.55
BIPH	0.39	3.36	1.46
CDNR	0.13	0.87	0.38
CTHR	-0.01	0.45	0.20
DSTI	0.85	2.05	0.89
DXPE	1.58	-0.53	-0.23
ELRN	0.72	-0.85	-0.37
GGBM	2.43	-1.65	-0.72
HCO	-1.36	2.30	1.00
ICOP	1.90	-2.46	-1.07

PROBABILITY OUTPUT

Percentile	*Y*
1.85	-0.97
5.56	-0.91
9.26	-0.79
12.96	-0.56
16.67	-0.46
20.37	-0.35
24.07	-0.32
27.78	-0.28
31.48	-0.26
35.19	-0.21
38.89	-0.13

IGLD	1.13	-1.39	-0.60		42.59	-0.12
INMD	-0.22	2.66	1.16		46.30	-0.06
ISR	0.74	1.32	0.58		50.00	0.04
JADE	0.87	0.29	0.13		53.70	0.45
JYSR	0.75	0.19	0.08		57.41	0.79
LMRA	1.38	-1.73	-0.75		61.11	0.94
LTRX	0.19	-0.99	-0.43		64.81	0.95
MRY	0.49	-0.81	-0.35		68.52	1.01
PEIX	2.95	-2.92	-1.27		72.22	1.05
PRH	0.55	-1.46	-0.63		75.93	1.16
SURG	0.81	-1.02	-0.44		79.63	1.23
TASR	3.94	8.19	3.56		83.33	2.07
TKO	1.06	0.17	0.07		87.04	2.44
VYHN	-0.01	-0.28	-0.12		90.74	2.90
WAVX	1.13	-2.10	-0.91		94.44	3.75
NASDAQ	-0.01	-0.05	-0.02		98.15	12.13

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Duke University has both a two-asset and a multi-asset efficient frontier generator online:

- Two-asset: http://www.duke.edu/~charvey/twoasset/index.html
- Multi-asset: http://www.duke.edu/~charvey/frontier/frontier.html

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