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Abstract: An important role of accountants is to provide the financial summary for the economic activities of companies over the period. The preparation of financial statements is nothing but a linear aggregation process of accounting information. The numbers summarized in financial statements balance (i.e. aggregated financial information) computed from the aggregation process of a myriad of day-to-day transactions (i.e. disaggregated financial information) of companies. The consumers of accounting information utilize aggregated or disaggregated accounting information in their decision making. Discriminant analysis is a pervasive field of which accounting information is used to separate distinct sets of entities. This paper provide practices of examining the effects of aggregation by examining the impacts of aggregation on the discriminating between two entities. In addition, the students who take financial accounting courses can be benefited by understanding the accounting nature of linear procedures.

Key words: Aggregation, Discriminant Analysis, Unequal variance

JEL classification: M40, M41, M49

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[1]Preliminary version

1 Introduction

An important role of accountants is to provide the financial summary to decision makers pertaining to the economic activities of companies over the period. The preparation of financial statements is nothing but a linear process of accounting information aggregation. The numbers summarized in financial statements balance (i.e. aggregated financial information) computed from the aggregation process of a myriad of day-to-day transactions (i.e. disaggregated financial information) of the companies. Accounting system collects and processes financial information of companies, and summaries and reports comprehensive information in relatively few line items. For example, financial accounting collects and processes information regarding daily transactions between the company and suppliers. Accountants aggregates those transactions and report in accounting balances including account payables, cash, or inventory. Managerial accounting aggregates various product costs into cost of good sold.

Discriminant analysis is applied to separate distinct sets of entities. In particular, this study investigate the relationship between discrimination and accounting aggregation procedure. Various applications can follow the discriminant analysis. For example, an auditor can use discriminant analysis in evaluating financial statements of audit clients (Koh and Killough, 1990). Bankruptcy prediction is another pervasive theme in applying the discriminant analysis to business (Altman, 1968; Balcaen and Ooghe, 2006). Banks can determine whether a firm should be classified as high credit risk or low credit risk using financial statement(i.e. aggregated accounting information). In doing so, they can also estimate the costs of misclassifying the entity. Explicit costs would be attached to the misclassifying an entity. A bank approves a loan to a firm by incorrectly classifying the company as low credit risk increases the likelihood that it would suffer from the potential loss due to the default of the company. On the other hand, a bank rejects a loan to a firm by incorrectly classifying the company as high credit is subject to the potential loss of profit opportunity. Therefore, accurate discrimination process is of substantial importance to various stakeholders.

An optimal decision rule for the discrimination is to minimize the average or *expected cost of misclassification* (ECM). Aforementioned example shows two types of errors are associated with ECM. Classifying a firm as not likely to default when it does default is *Type I error.* On the other hand, Classifying a firm as likely to default when it does not default is *Type II error.* It is assumed that the decision makers adopt the classification scheme evaluated in terms of ECM.

Arya et al. (2000) examines the costs and benefits of aggregating financial information in the discriminating between two entities. They analyze the model in the context that the two entities fundamentally differ in their business activities. That is, the fundamental assumption of the previous research is that two entities have a common covariance matrix in transaction but have different mean value of their transaction matrices. While the equal covariance case uses linear discriminant function for the discrimination, unequal covariance case utilizes quadratic function. One purpose of this paper is providing accounting students with better understanding of accounting aggregation through the linear procedure. In the next section, the general discriminant model with unequal covariance matrices will be derived.

2 Basic Model of Classification

I assume that all parameters are known and I will follow linear procedures. Let π_1 and π_2 be $N(\mu_1, \Sigma_1)$ and $N(\mu_2, \Sigma_2)$ with $\mu_1 \neq \mu_2$ and $\Sigma_1 \neq \Sigma_2$, since the case $\Sigma_1 = \Sigma_2$ has been treated in the seminar. I assume that Σ_1 and Σ_2 are nonsingular.

Notation 1 *Let* $\mathbf{b} \neq \mathbf{0}$ *be a vector of* p *components and* c *be a scalar.*

Notation 2 $\Phi(z) = \int_{-\infty}^{z} (2\pi)^{-\frac{1}{2}} e^{-\frac{1}{2}t^2} dt$

An observation $\mathbf{x}$ is classified as from π_1 if $\mathbf{b}'\mathbf{x} \leq c$ and as from π_2 if $\mathbf{b}'\mathbf{x} > c$. $\mathbf{b}'_{(\mathbf{1}\times\mathbf{p})}\mathbf{x}_{(\mathbf{p}\times\mathbf{1})}$ is a univariate normal distribution.

The mean is

$$\mathrm{E}(\mathbf{b}'\mathbf{x}) = \mathbf{b}'\mu_{\mathbf{i}}, \qquad i = 1, 2 \tag{1}$$

The variance is

$$
\begin{aligned}
\mathrm{E}(\mathbf{b}'\mathbf{x} - \mathbf{b}'\mu_{\mathbf{i}})^2 &= \mathrm{E}(\mathbf{b}'(\mathbf{x} - \mu_{\mathbf{i}})(\mathbf{x} - \mu_{\mathbf{i}})'\mathbf{b}) \\
&= \mathbf{b}'\mathbf{\Sigma_i}\mathbf{b}, \qquad i = 1, 2
\end{aligned} \tag{2}
$$

3 Benchmark Solutions

$$
\begin{aligned}
P(2 \mid 1) &= \Pr\{\mathbf{b}'\mathbf{x} > c \mid \pi_1\} \\
&= \Pr\left\{ \frac{\mathbf{b}'\mathbf{x} - \mathbf{b}'\mu_{\mathbf{1}}}{(\mathbf{b}'\mathbf{\Sigma_1}\mathbf{b})^{\frac{1}{2}}} > \frac{c - \mathbf{b}'\mu_{\mathbf{1}}}{(\mathbf{b}'\mathbf{\Sigma_1}\mathbf{b})^{\frac{1}{2}}} \middle| \pi_1 \right\} \\
&= 1 - \Phi\left(\frac{c - \mathbf{b}'\mu_{\mathbf{1}}}{(\mathbf{b}'\mathbf{\Sigma_1}\mathbf{b})^{\frac{1}{2}}} \right)
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
P(1 \mid 2) &= \Pr\{\mathbf{b}'\mathbf{x} \leq c \mid \pi_2\} \\
&= \Pr\left\{ \frac{\mathbf{b}'\mathbf{x} - \mathbf{b}'\mu_{\mathbf{2}}}{(\mathbf{b}'\mathbf{\Sigma_2}\mathbf{b})^{\frac{1}{2}}} \leq \frac{c - \mathbf{b}'\mu_{\mathbf{2}}}{(\mathbf{b}'\mathbf{\Sigma_2}\mathbf{b})^{\frac{1}{2}}} \middle| \pi_2 \right\} \\
&= 1 - \Phi\left(\frac{\mathbf{b}'\mu_{\mathbf{2}} - c}{(\mathbf{b}'\mathbf{\Sigma_2}\mathbf{b})^{\frac{1}{2}}} \right)
\end{aligned} \tag{4}
$$

I want to minimize these two probabilities. In other words, I desire to maximize following arguments

$$
y_1 = \frac{c - \mathbf{b}'\mu_{\mathbf{1}}}{(\mathbf{b}'\mathbf{\Sigma_1}\mathbf{b})^{\frac{1}{2}}} \tag{5}
$$

$$
y_2 = \frac{\mathbf{b}'\mu_{\mathbf{2}} - c}{(\mathbf{b}'\mathbf{\Sigma_2}\mathbf{b})^{\frac{1}{2}}} \tag{6}
$$

From the equation (6),

$$
c = \mathbf{b}'\mu_{\mathbf{2}} - y_2(\mathbf{b}'\mathbf{\Sigma_2}\mathbf{b})^{\frac{1}{2}} \tag{7}
$$

Then

$$
y_1 = \frac{\mathbf{b}'\mu_{\mathbf{2}} - \mathbf{b}'\mu_{\mathbf{1}} - y_2(\mathbf{b}'\mathbf{\Sigma_2}\mathbf{b})^{\frac{1}{2}}}{(\mathbf{b}'\mathbf{\Sigma_1}\mathbf{b})^{\frac{1}{2}}}
$$

Let $\delta = \mu_{\mathbf{2}} - \mu_{\mathbf{1}}$

$$
y_1 = \frac{\mathbf{b}'\delta - y_2(\mathbf{b}'\mathbf{\Sigma_2}\mathbf{b})^{\frac{1}{2}}}{(\mathbf{b}'\mathbf{\Sigma_1}\mathbf{b})^{\frac{1}{2}}} \tag{8}
$$

I differentiate y_1 with respect to b to maximize y_1 given y_2.

$$\begin{aligned}
\frac{\partial y_1}{\partial b} &= \left(\delta - \frac{1}{2} y_2 (\mathbf{b}'\mathbf{\Sigma_2 b})^{-\frac{1}{2}}\, 2\,\Sigma_2 \mathbf{b}\right)(\mathbf{b}'\mathbf{\Sigma_1 b})^{-\frac{1}{2}} \\
&\qquad - \frac{1}{2}(\mathbf{b}'\mathbf{\Sigma_1 b})^{-\frac{3}{2}}(\mathbf{b}'\delta - y_2(\mathbf{b}'\mathbf{\Sigma_2 b})^{\frac{1}{2}})2\,\mathbf{\Sigma_1 b} \\
&= \delta(\mathbf{b}'\mathbf{\Sigma_1 b})^{-\frac{1}{2}} - y_2(\mathbf{b}'\mathbf{\Sigma_2 b})^{-\frac{1}{2}}\mathbf{\Sigma_2 b}(\mathbf{b}'\mathbf{\Sigma_1 b})^{-\frac{1}{2}} \\
&\qquad - \mathbf{b}'\delta(\mathbf{b}'\mathbf{\Sigma_2 b})^{-\frac{3}{2}}\mathbf{\Sigma_1 b} + y_2(\mathbf{b}'\mathbf{\Sigma_2 b})^{\frac{1}{2}}(\mathbf{b}'\mathbf{\Sigma_1 b})^{-\frac{3}{2}}\mathbf{\Sigma_1 b} \\
&= (\mathbf{b}'\mathbf{\Sigma_1 b})^{-\frac{1}{2}}\Big[\delta - y_2(\mathbf{b}'\mathbf{\Sigma_2 b})^{-\frac{1}{2}}\mathbf{\Sigma_2 b} - \mathbf{b}'\delta(\mathbf{b}'\mathbf{\Sigma_1 b})^{-1}\mathbf{\Sigma_1 b} \\
&\qquad + y_2\,(\mathbf{b}'\mathbf{\Sigma_2 b})^{\frac{1}{2}}(\mathbf{b}'\mathbf{\Sigma_1 b})^{-1}\mathbf{\Sigma_1 b}\Big] = 0
\end{aligned}$$

Since $(\mathbf{b}'\mathbf{\Sigma_1 b})^{-\frac{1}{2}}$ is positive definite

$$\Big[\delta - y_2(\mathbf{b}'\mathbf{\Sigma_2 b})^{-\frac{1}{2}}\mathbf{\Sigma_2 b} - \mathbf{b}'\delta(\mathbf{b}'\mathbf{\Sigma_1 b})^{-1}\mathbf{\Sigma_1 b} + y_2(\mathbf{b}'\mathbf{\Sigma_2 b})^{\frac{1}{2}}(\mathbf{b}'\mathbf{\Sigma_1 b})^{-1}\mathbf{\Sigma_1 b}\Big] = 0$$

$$\begin{aligned}
\delta &= y_2(\mathbf{b}'\mathbf{\Sigma_2 b})^{-\frac{1}{2}} + \mathbf{b}'\delta(\mathbf{b}'\mathbf{\Sigma_1 b})^{-1}\mathbf{\Sigma_1 b} - y_2(\mathbf{b}'\mathbf{\Sigma_2 b})^{\frac{1}{2}}(\mathbf{b}'\mathbf{\Sigma_1 b})^{-1}\mathbf{\Sigma_1 b} \\
&= \left[\left(\frac{y_2}{(\mathbf{b}'\mathbf{\Sigma_2 b})^{\frac{1}{2}}}\right)\mathbf{\Sigma_2} + \left(\frac{\mathbf{b}'\delta - y_2(\mathbf{b}'\mathbf{\Sigma_2 b})^{\frac{1}{2}}}{\mathbf{b}'\mathbf{\Sigma_1 b}}\right)\mathbf{\Sigma_1}\right]\mathbf{b} \qquad (9)
\end{aligned}$$

Let

$$t_1 = \frac{\mathbf{b}'\delta - y_2(\mathbf{b}'\mathbf{\Sigma_2 b})^{\frac{1}{2}}}{\mathbf{b}'\mathbf{\Sigma_1 b}} \qquad (10)$$

$$t_2 = \frac{y_2}{(\mathbf{b}'\mathbf{\Sigma_2 b})^{\frac{1}{2}}} \qquad (11)$$

Then

$$\delta = (t_1\mathbf{\Sigma_1} + t_2\mathbf{\Sigma_2})\mathbf{b} \qquad (12)$$

From (5) and (11)

$$\begin{aligned}
c &= \mathbf{b}'\mu_{\mathbf{2}} - y_2(\mathbf{b}'\mathbf{\Sigma_2 b})^{\frac{1}{2}} \\
&= \mathbf{b}'\mu_{\mathbf{2}} - t_2\mathbf{b}'\mathbf{\Sigma_2 b} \qquad (13)
\end{aligned}$$

From (8) and (10)

$$\begin{aligned}
y_1 &= \frac{\mathbf{b}'\delta - y_2(\mathbf{b}'\mathbf{\Sigma_2 b})^{\frac{1}{2}}}{(\mathbf{b}'\mathbf{\Sigma_1 b})^{\frac{1}{2}}} \\
&= t_1(\mathbf{b}'\mathbf{\Sigma_1 b})^{\frac{1}{2}} \qquad (14)
\end{aligned}$$

From (11)

$$y_2 = t_2(\mathbf{b}'\mathbf{\Sigma_2}\mathbf{b})^{\frac{1}{2}} \tag{15}$$

Note that the right hand sides of (14) and (15) are homogeneous of degree 0 in t_1 and t_2. In other words, if I plug (12) in (14) and (15), I always get same values in y_1 and y_2 regardless of t_1 and t_2. Therefore it is convenient if I normalize t_1 and t_2 such that

$$t_1 + t_2 = 1$$

If I can show that y_1 is a monotonic increasing function of t_1 and y_2 is a monotonic decreasing function of $t_1(0 \leq t_1 \leq 1)$, I can calculate optimum $\mathbf{b}$ (I will discuss this later).

Since $\mathbf{\Sigma_1}$ and $\mathbf{\Sigma_2}$ are positive definite matrices, I can use *Cholesky decomposition.* In other words, there exists a matrix $\mathbf{R}$ with independent columns. For the convenience, I can transform the covariance matrices to the following form

$$\mathbf{\Sigma_2} = \mathbf{R}'\mathbf{R},$$

$$\mathbf{\Sigma_1} = \mathbf{R}'\mathbf{\Lambda}\mathbf{R} = \mathbf{R}'\begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_p \end{pmatrix}\mathbf{R}, \qquad \lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_p > 0$$

$$\delta = \mathbf{R}'\theta,$$

From (12)

$$\mathbf{b} = (t_1\mathbf{\Sigma_1} + t_2\mathbf{\Sigma_2})^{-1}\delta$$

$$\mathbf{b}' = \delta'(t_1\mathbf{\Sigma_1} + t_2\mathbf{\Sigma_2})^{-1'}$$

Then

$$
\begin{aligned}
y_1 &= t_1(\mathbf{b}'\mathbf{\Sigma_1}\mathbf{b})^{\frac{1}{2}} \\
&= t_1[\delta'(t_1\mathbf{\Sigma_1}+t_2\mathbf{\Sigma_2})^{-1'}\mathbf{\Sigma_1}(t_1\mathbf{\Sigma_1}+t_2\mathbf{\Sigma_2})^{-1}\delta]^{\frac{1}{2}} \\
&= t_1[\theta'\mathbf{R}((t_1\mathbf{\Sigma_1})'+(t_2\mathbf{\Sigma_2})')^{-1}\mathbf{\Sigma_1}(t_1\mathbf{\Sigma_1}+t_2\mathbf{\Sigma_2})^{-1}\mathbf{R}'\theta]^{\frac{1}{2}} \\
&= t_1[\theta'\mathbf{R}(\mathbf{\Sigma_1}'t_1+\mathbf{\Sigma_2}'t_2)^{-1}\mathbf{\Sigma_1}(t_1\mathbf{\Sigma_1}+t_2\mathbf{\Sigma_2})^{-1}\mathbf{R}'\theta]^{\frac{1}{2}} \\
&= t_1[\theta'\mathbf{R}(\mathbf{R}'\mathbf{\Lambda}\mathbf{R}t_1+\mathbf{R}'\mathbf{I}\mathbf{R}t_2)^{-1}\mathbf{R}'\mathbf{\Lambda}\mathbf{R}(t_1\mathbf{R}'\mathbf{\Lambda}\mathbf{R}+t_2\mathbf{R}'\mathbf{I}\mathbf{R})^{-1}\mathbf{R}'\theta]^{\frac{1}{2}} \\
&= t_1[\theta'\mathbf{R}(\mathbf{R}'(\mathbf{\Lambda}t_1+\mathbf{I}t_2)\mathbf{R})^{-1}\mathbf{R}'\mathbf{\Lambda}\mathbf{R}(\mathbf{R}'(t_1\mathbf{\Lambda}+t_2\mathbf{I})\mathbf{R})^{-1}\mathbf{R}'\theta]^{\frac{1}{2}} \\
&= t_1[\theta'\mathbf{R}\mathbf{R}^{-1}(\mathbf{\Lambda}t_1+\mathbf{I}t_2)^{-1}\mathbf{R}'^{-1}\mathbf{R}'\mathbf{\Lambda}\mathbf{R}\mathbf{R}^{-1}(t_1\mathbf{\Lambda}+t_2\mathbf{I})^{-1}\mathbf{R}'^{-1}\mathbf{R}'\theta]^{\frac{1}{2}} \\
&= t_1[\theta'(\mathbf{\Lambda}t_1+\mathbf{I}t_2)^{-1}\mathbf{\Lambda}(\mathbf{\Lambda}t_1+\mathbf{I}t_2)^{-1}\theta]^{\frac{1}{2}} \\
&= t_1\left[\sum_{i=1}^{p}\frac{\theta_{\mathbf{i}}^{\mathbf{2}}\lambda_{\mathbf{i}}}{(t_1\lambda_{\mathbf{i}}+t_2)^2}\right]^{\frac{1}{2}} \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
y_2 &= t_2(\mathbf{b}'\mathbf{\Sigma_2}\mathbf{b})^{\frac{1}{2}} \\
&= t_2[\delta'(t_1\mathbf{\Sigma_1}+t_2\mathbf{\Sigma_2})^{-1'}\mathbf{\Sigma_2}(t_1\mathbf{\Sigma_1}+t_2\mathbf{\Sigma_2})^{-1}\delta]^{\frac{1}{2}} \\
&= t_2[\theta'\mathbf{R}((t_1\mathbf{\Sigma_1})'+(t_2\mathbf{\Sigma_2})')^{-1}\mathbf{\Sigma_2}(t_1\mathbf{\Sigma_1}+t_2\mathbf{\Sigma_2})^{-1}\mathbf{R}'\theta]^{\frac{1}{2}} \\
&= t_2[\theta'\mathbf{R}(\mathbf{\Sigma_1}'t_1+\mathbf{\Sigma_2}'t_2)^{-1}\mathbf{\Sigma_2}(t_1\mathbf{\Sigma_1}+t_2\mathbf{\Sigma_2})^{-1}\mathbf{R}'\theta]^{\frac{1}{2}} \\
&= t_2[\theta'\mathbf{R}(\mathbf{R}'\mathbf{\Lambda}\mathbf{R}t_1+\mathbf{R}'\mathbf{I}\mathbf{R}t_2)^{-1}\mathbf{R}'\mathbf{I}\mathbf{R}(t_1\mathbf{R}'\mathbf{\Lambda}\mathbf{R}+t_2\mathbf{R}'\mathbf{I}\mathbf{R})^{-1}\mathbf{R}'\theta]^{\frac{1}{2}} \\
&= t_2[\theta'\mathbf{R}(\mathbf{R}'(\mathbf{\Lambda}t_1+\mathbf{I}t_2)\mathbf{R})^{-1}\mathbf{R}'\mathbf{I}\mathbf{R}(\mathbf{R}'(t_1\mathbf{\Lambda}+t_2\mathbf{I})\mathbf{R})^{-1}\mathbf{R}'\theta]^{\frac{1}{2}} \\
&= t_2[\theta'\mathbf{R}\mathbf{R}^{-1}(\mathbf{\Lambda}t_1+\mathbf{I}t_2)^{-1}\mathbf{R}'^{-1}\mathbf{R}'\mathbf{I}\mathbf{R}\mathbf{R}^{-1}(t_1\mathbf{\Lambda}+t_2\mathbf{I})^{-1}\mathbf{R}'^{-1}\mathbf{R}'\theta]^{\frac{1}{2}} \\
&= t_2[\theta'(\mathbf{\Lambda}t_1+\mathbf{I}t_2)^{-1}(\mathbf{\Lambda}t_1+\mathbf{I}t_2)^{-1}\theta]^{\frac{1}{2}} \\
&= t_2\left[\sum_{i=1}^{p}\frac{\theta_{\mathbf{i}}^{\mathbf{2}}}{(t_1\lambda_{\mathbf{i}}+t_2)^2}\right]^{\frac{1}{2}} \qquad (17)
\end{aligned}
$$

Since $y_1 > 0$ and $y_2 > 0$, it is convenient to take the derivative of y_1^2 instead of y_1 where $t_2 = 1 - t_1$.

$$
\begin{aligned}
y_1^2 &= t_1^2\sum_{i=1}^{p}\frac{\theta_{\mathbf{i}}^{\mathbf{2}}\lambda_{\mathbf{i}}}{(t_1\lambda_{\mathbf{i}}+t_2)^2} \\
&= t_1^2\frac{\theta_{\mathbf{1}}^{\mathbf{2}}\lambda_{\mathbf{1}}}{(t_1\lambda_{\mathbf{1}}+1-t_1)^2}+\cdots
\end{aligned}
$$

$$\begin{aligned} y_2^2 &= t_2^2 \sum_{i=1}^{p} \frac{\theta_{\mathbf{i}}^{\mathbf{2}}}{(t_1\lambda_{\mathbf{i}} + t_2)^2} \\ &= t_2^2 \frac{\theta_{\mathbf{1}}^{\mathbf{2}}}{(t_1\lambda_{\mathbf{1}} + 1 - t_1)^2} + \cdots \end{aligned}$$

$$\begin{aligned} \frac{\partial y_1^2}{\partial t_1} &= \frac{2t_1\theta_{\mathbf{1}}^{\mathbf{2}}\lambda_{\mathbf{1}}(t_1\lambda_{\mathbf{1}} + 1 - t_1)^2 - 2(t_1\lambda_{\mathbf{1}} + 1 - t_1)(\lambda_{\mathbf{1}} - 1)t_1^2\theta_{\mathbf{1}}^{\mathbf{2}}\lambda_{\mathbf{1}}}{(t_1\lambda_{\mathbf{1}} + 1 - t_1)^4} + \cdots \\ &= \frac{2t_1\theta_{\mathbf{1}}^{\mathbf{2}}\lambda_{\mathbf{1}}}{(t_1\lambda_{\mathbf{1}} + 1 - t_1)^3} + \cdots \\ &= 2t_1 \sum_{i=1}^{p} \frac{\theta_{\mathbf{i}}^{\mathbf{2}}\lambda_{\mathbf{i}}}{(t_1\lambda_{\mathbf{1}} + 1 - t_1)^3} > 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial y_2^2}{\partial t_1} &= \frac{-2(1 - t_1)\theta_{\mathbf{1}}^{\mathbf{2}}(t_1\lambda_{\mathbf{1}} + 1 - t_1)^2 - 2(t_1\lambda_{\mathbf{1}} + 1 - t_1)(\lambda_{\mathbf{1}} - 1)(1 - t_1)^2\theta_{\mathbf{1}}^{\mathbf{2}}}{(t_1\lambda_{\mathbf{1}} + 1 - t_1)^4} + \cdots \\ &= \frac{-2(1 - t_1)\theta_{\mathbf{1}}^{\mathbf{2}}\lambda_{\mathbf{1}}}{(t_1\lambda_{\mathbf{1}} + 1 - t_1)^3} + \cdots \\ &= -2(1 - t_1) \sum_{i=1}^{p} \frac{\theta_{\mathbf{i}}^{\mathbf{2}}\lambda_{\mathbf{i}}}{(t_1\lambda_{\mathbf{1}} + 1 - t_1)^3} < 0 \end{aligned}$$

Therefore, y_1 is a monotonic increasing function of t_1 and y_2 is a monotonic decreasing function of $t_1 (0 \leq t_1 \leq 1)$.

4 Use of Transaction Matrices

The double entry transformation matrix A and the six transactions are

$$\begin{cases} t_1: & \text{collections of accounts receivable} \\ t_2: & \text{cash purchase of inventory} \\ t_3: & \text{credit sales} \\ t_4: & \text{cost of goods sold recognized} \\ t_5: & \text{cash sales} \\ t_6: & \text{cash expenses} \end{cases}$$

$$\begin{array}{c c l} & \begin{array}{cccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 \end{array} & \\ \mathbf{A} = & \begin{pmatrix} 1 & -1 & 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} & \begin{array}{c} \text{Cash} \\ \text{A/R} \\ \text{Inventory} \\ \text{Sales} \\ \text{Expenses} \end{array} \end{array}$$

The covariance matrices are

$$\mathbf{\Sigma_1} = \begin{pmatrix} 0.3 & 0.1 & 0.2 & 0 & 0 & 0 \\ 0 & 1.4 & 0 & 0.4 & 0 & 0 \\ 0.1 & 0 & 1.2 & 0 & 0 & 0.3 \\ 0.1 & 0 & 0 & 0.9 & 0 & 0 \\ 0 & 0.3 & 0 & 0.3 & 1.0 & 0 \\ 0.2 & 0 & 0.5 & 0 & 0 & 4 \end{pmatrix}$$

$$\mathbf{\Sigma_2} = \begin{pmatrix} 0.4 & 0.2 & 0.3 & 0 & 0 & 0 \\ 0 & 1.7 & 0 & 0.4 & 0 & 0 \\ 0 & 0 & 1.7 & 0 & 0.4 & 0 \\ 0.1 & 0 & 0 & 1.3 & 0 & 0 \\ 0.2 & 0.3 & 0 & 0 & 1.4 & 0 \\ 0.6 & 0.5 & 0 & 0 & 0 & 0.8 \end{pmatrix}$$

Suppose the mean transaction matrices are

$$\mu_{\mathbf{1}} = \begin{pmatrix} 4.25 & 5.25 & 4 & 5 & 1.5 \end{pmatrix}'$$
$$\mu_{\mathbf{2}} = \begin{pmatrix} 4.25 & 5.25 & 5 & 6 & 1.5 \end{pmatrix}'$$

If t_1 and t_2 are given, I can calculate optimal $\mathbf{b}$ by the equation (12). Then I can compute c by the equation (7). However, t_1 and t_2 are rarely known. So I should restrict our case to the following way.

4.1 Ether y_1 or y_2 is given

Suppose that y_2 is given. If $y_2 = y_2^\star$, then $y_2^\star = t_2(\mathbf{b}'\mathbf{\Sigma_2}\mathbf{b})^{\frac{1}{2}}$,where $\mathbf{b} = (t_1\mathbf{\Sigma_1} + t_2\mathbf{\Sigma_2})^{-1}\delta$. Therefore

$$y_2^\star = (1-t_1)\left[\delta'\{t_1\mathbf{\Sigma_1} + (1-t_1)\mathbf{\Sigma_2}\}^{-1'}\mathbf{\Sigma_2}\{t_1\mathbf{\Sigma_1} + (1-t_1)\mathbf{\Sigma_2}\}^{-1}\delta\right]^{\frac{1}{2}} \tag{18}$$

Since $y_2^\star$ is known and y_2 is a decreasing function of t_1, I can easily approximate t_1 by trial and error($0 < t_1 \leq 1$).

Now I can compute $\mathbf{b} = \{t_1\mathbf{\Sigma_1} + (1-t_1)\mathbf{\Sigma_2}\}^{-1}\delta$. This is minimizing misclassification error.

Suppose $p(1|2)$ is known to be 46%. In other words, $\Phi(y_2) = 1 - p(1|2)$ and $y_2^\star = 0.1$. Since y_2 is decreasing function of t_1, I can try other values of t_1 and insert in the equation (18) until I get the value $y_2 \approx y_2^\star$. I can compute $t_1 \approx 0.93$ by trial and error. Therefore, the optimum vector $\mathbf{b}$ is computed by (12).

$$\mathbf{b} = \begin{pmatrix} 0.157 & -0.258 & -0.0796 & 0.961 & 0.772 & 0.00543 \end{pmatrix}'$$

$$y_1 = 0.1 \qquad \text{by (5)}$$

Therefore, the probability of misclassification $p(2|1)$ is

$$p(2|1) = 1 - \Phi(y1) = 1 - 0.89435 = 0.10565$$

t_1	y_2	y_1	$p(1\|2)$	$p(2\|1)$	$p(1\|2) + p(2\|1)$
0.930	0.1000	1.2500	0.46017	0.10565	0.56582
0.860	0.2000	1.1300	0.42074	0.12924	0.54998
0.780	0.3000	1.0100	0.38209	0.15625	0.53834
0.720	0.4004	0.9193	0.34443	0.17897	0.52340
0.640	0.5047	0.8023	0.30688	0.21119	0.51807
0.610	0.5428	0.7595	0.29363	0.22378	0.51741
0.600	0.5554	0.7454	0.28931	0.22801	**0.51733**
0.590	0.5679	0.7313	0.28505	0.23230	0.51735
0.560	0.6051	0.6895	0.27256	0.24525	0.51781
0.550	0.6173	0.6756	0.26852	0.24965	0.51817
0.540	0.6296	0.6619	0.26448	0.25402	0.51850
0.528	0.6442	0.6455	0.25972	0.25930	0.51902
0.528	**0.6448**	**0.6448**	0.25953	0.25953	**0.51906**
0.480	0.7000	0.5800	0.24196	0.28096	0.52292
0.400	0.8000	0.4800	0.21186	0.31561	0.52747
0.300	0.9000	0.3500	0.18406	0.36317	0.54723
0.210	1.0000	0.2400	0.15866	0.40517	0.56382

4.2 Minimax procedure

Suppose $y_1 = y_2$. This equality is same as $y_1^2 = y_2^2$ because $y_1 > 0$ and $y_2 > 0$.

$$\begin{aligned} 0 = y_1^2 - y_2^2 &= t_1^2 \mathbf{b}'\boldsymbol{\Sigma_1}\mathbf{b} - (1-t_1)^2 \mathbf{b}'\boldsymbol{\Sigma_2}\mathbf{b} \\ &= \mathbf{b}'[t_1^2\boldsymbol{\Sigma_1} - (1-t_1)^2\boldsymbol{\Sigma_2}]\mathbf{b} \end{aligned}$$

In this case, I can guess a value of t_1 and solve the quadratic equation for $\mathbf{b}$. I get $t_1 \approx 0.5275$.

$$y_1 = y_2 = 0.6448 \qquad \text{by (5) and (6)}$$

Therefore, the probability of misclassification $p(1|2)$ and $p(2|1)$ are

$$p(1|2) = p(2|1) = 1 - \Phi(y1) = 1 - 0.74047 = 0.25953$$

4.3 Case of a priori probabilities and cost function

If I are given a priori probabilities, p_1 and p_2, and the cost functions, $c(1|2)$ and $c(2|1)$, the probability of a misclassification is

$$p_1 c(2|1)[1-\Phi(y_1)] + p_2 c(1|2)[1-\Phi(y_2)]$$

If I take derivative of the equation above

$$p_1 c(2|1)\Phi(y_1)\frac{\partial y_1}{\partial t_1} + p_2 c(1|2)\Phi(y_2)\frac{\partial y_2}{\partial t_1} = 0 \tag{19}$$

There is no easy solution to the differential equation (19).

4.3.1 $\Sigma_1 = k\Sigma_2$

$$\frac{\frac{\partial y_2}{\partial t_1}}{\frac{\partial y_1}{\partial t_1}} = -\frac{(\mathbf{b}'\mathbf{\Sigma_2 b})^{-\frac{1}{2}}}{(\mathbf{b}'\mathbf{\Sigma_1 b})^{-\frac{1}{2}}} \qquad \text{by the Envelop theorems 1994}$$

Therefore, the equation (19) can be expressed as

$$\begin{aligned}
\frac{p_1 c(2|1)}{(\mathbf{b}'\mathbf{\Sigma_1 b})^{\frac{1}{2}}}\Phi(y_1) &= \frac{p_2 c(1|2)}{(\mathbf{b}'\mathbf{\Sigma_2 b})^{\frac{1}{2}}}\Phi(y_2) \qquad (20)\\
\frac{p_1 c(2|1)}{(\mathbf{b}'\mathbf{\Sigma_1 b})^{\frac{1}{2}}}\Phi(y_1) &= \frac{p_2 c(1|2)}{\frac{1}{\sqrt{k}}(\mathbf{b}'\mathbf{\Sigma_2 b})^{\frac{1}{2}}}\Phi(y_2)\\
\phi(y_1) &= \frac{p_2 c(1|2)}{p_1 c(2|1)}\sqrt{k}\phi(y_2)\\
(2\pi)^{-\frac{1}{2}}e^{-\frac{1}{2}y_1^2} &= \frac{p_2 c(1|2)}{p_1 c(2|1)}\sqrt{k}(2\pi)^{-\frac{1}{2}}e^{-\frac{1}{2}y_2^2}\\
e^{-\frac{1}{2}(t_1^2(\mathbf{b}'\mathbf{\Sigma_1 b})-(1-t_1)^2(\mathbf{b}'\mathbf{\Sigma_2 b}))} &= \frac{p_2 c(1|2)}{p_1 c(2|1)}\sqrt{k}\\
-\frac{1}{2}(t_1^2(\mathbf{b}'\mathbf{\Sigma_1 b}) - \frac{(1-t_1)^2}{k}(\mathbf{b}'\mathbf{\Sigma_1 b})) &= \ln\left(\frac{p_2 c(1|2)}{p_1 c(2|1)}\right) + \frac{1}{2}\ln k\\
(-kt_1^2 + (1-t_1)^2)\mathbf{b}'\mathbf{\Sigma_1 b} &= k\ln k + 2k\ln\left(\frac{p_2 c(1|2)}{p_1 c(2|1)}\right)\\
((1-t_1)^2 - (\sqrt{k}\,t_1)^2)\mathbf{b}'\mathbf{\Sigma_1 b} &= k\ln k + 2k\ln\left(\frac{p_2 c(1|2)}{p_1 c(2|1)}\right)\\
(1-t_1-\sqrt{k}t_1)(1-t_1+\sqrt{k}t_1)\mathbf{b}'\mathbf{\Sigma_1 b} &= k\ln k + 2k\ln\left(\frac{p_2 c(1|2)}{p_1 c(2|1)}\right) \qquad (21)
\end{aligned}$$

As I proved in (16) and (17), $\mathbf{b}'\mathbf{\Sigma_1 b}$ is a monotonic increasing function in t_1. Therefore, whether LHS of the equation is monotonic increasing or decreasing in t_1 depends on the sign of $(1-t_1-\sqrt{k}t_1)(1-t_1+\sqrt{k}t_1)\mathbf{b}'\mathbf{\Sigma_1}\mathbf{b}$. Since $0 \le t_1 \le 1$, I can consider the following cases.

$k < 1$	$\frac{1}{1+\sqrt{k}} < t_1 < 1$	Monotonic Decreasing
$k < 1$	$0 \le t_1 \le \frac{1}{1+\sqrt{k}}$	Monotonic Increasing
$k \ge 1$	$0 < t_1 < \frac{1}{1+\sqrt{k}}$	Monotonic Decreasing
$k \ge 1$	$\frac{1}{1+\sqrt{k}} \le t_1 \le 1$	Monotonic Increasing

Suppose $\mathbf{\Sigma_1} = 2\mathbf{\Sigma_2}$, $p_1 = p_2 = 0.5$, and $c(1|2) = c(2|1) = 1$. RHS of the equation (21) becomes $k \ln k = 1.386$. Since LHS is monotonic decreasing in t_1, I can easily compute t_1 by trial and error. In this case, $t_1 = 0.2986$, $y_1 = 0.63$ and $y_2 = 1.04$. As a result, the total cost of misclassification is

$$0.5(1-\Phi y_1) + 0.5(1-\Phi y_2) = 0.5(0.1492 + 0.2643) = 0.2063$$

There are ECM's for the different k's below.

k	y_2	y_1	$p(1\|2)$	$p(2\|1)$	$0.5\,p(1\|2) + 0.5\,p(2\|1)$
0.3	1.12	0.14	0.1314	0.4443	0.2878
0.5	0.90	0.32	0.1841	0.3745	0.2793
1.0	0.68	0.68	0.2483	0.2483	0.2483
1.5	0.52	0.89	0.3015	0.1867	0.2441
2.0	0.63	1.04	0.2643	0.1492	0.2068
3.0	0.65	1.23	0.2578	0.1093	0.1836
4.0	0.68	1.36	0.2483	0.0869	0.1676
5.0	0.71	1.45	0.2389	0.0735	0.1562

4.3.2 $\Sigma_1 = \Sigma_2$

I want to show the result of the linear procedure is consistant with the analysis described in the paper (Arya et al., 2000).

$$(1-t_1-\sqrt{k}t_1)(1-t_1+\sqrt{k}t_1)\mathbf{b}'\mathbf{\Sigma_1}\mathbf{b} = k \ln k + 2k \ln\left(\frac{p_2 c(1|2)}{p_1 c(2|1)}\right)$$

$$(1-2t_1)\mathbf{b}'\mathbf{\Sigma_1}\mathbf{b} = 0$$

Since $\mathbf{b'\Sigma_1 b}$ is positive definite matrix, $t_1 = \frac{1}{2}$. Therefore, if I plug t_1 in the equation (12),

$$\begin{aligned}\delta &= (t_1\mathbf{\Sigma_1} + t_2\mathbf{\Sigma_2})\mathbf{b} \\ &= (\frac{1}{2}\mathbf{\Sigma_1} + \frac{1}{2}\mathbf{\Sigma_2})\mathbf{b} \\ &= \frac{1}{2}(\mathbf{\Sigma_1} + \mathbf{\Sigma_1}) \\ &= \mathbf{\Sigma_1 b}\end{aligned}$$

$$\mathbf{b} = \mathbf{\Sigma^{-1}}\delta \tag{22}$$

This vector is identical to the linear discriminant $\mathbf{l_y} = \mathbf{\Sigma_y^{-1}}\eta_{\mathbf{d}}$ (Arya et al., 2000).

5 Use of Balance Matrices

It is inevitable to lose information during the aggregation process since balance vector $\mathbf{x}$ is shorter than transaction vector $\mathbf{y}$ (i.e. $m \leq n$).

If I use the financial statement vector $\mathbf{x}$ instead of the transaction vector $\mathbf{y}$,

$$\mathbf{Ay} = \mathbf{x}$$

$$\mathbf{A} = \begin{array}{c} \begin{array}{cccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 \end{array} \\ \begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \end{array} \begin{array}{c} \\ \text{A/R} \\ \text{Inventory} \\ \text{Sales} \\ \text{Expenses} \end{array}$$

$$\mathbf{\Sigma_{x1}} = \mathbf{A\Sigma_1 A'}$$

$$\mathbf{\Sigma_{x2}} = \mathbf{A\Sigma_2 A'}$$

$$\mathbf{d} = \mathbf{A}\delta \tag{23}$$

$$\mathbf{b_x} = (s_1\mathbf{\Sigma_{x1}} + s_2\mathbf{\Sigma_{x2}})^{-1}\mathbf{d} \qquad 0 \leq s_1, s_2 \leq 1$$

$$\mathbf{x_1} = s_1(\mathbf{b'_x\Sigma_{x1}b_x})^{\frac{1}{2}} \tag{24}$$

$$\mathbf{x_2} = s_2(\mathbf{b'_x\Sigma_{x2}b_x})^{\frac{1}{2}} \tag{25}$$

Now I want to show that x_1 is a monotonic increasing function of s_1 and x_2 is a monotonic decreasing function of $s_1 (0 \leq t_1 \leq 1)$. Since $\mathbf{\Sigma_1}$ and $\mathbf{\Sigma_2}$ are positive definite matrices, there exists a rectangular matrix $\mathbf{G_{4\times 6}}$. For the convenience, I can transform the covariance matrices to the following form

$$\mathbf{\Sigma_2} = \mathbf{G}'\mathbf{G},$$

$$\mathbf{\Sigma_1} = \mathbf{G}'\mathbf{\Lambda}\mathbf{G} = \mathbf{G}'\begin{pmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{pmatrix}\mathbf{G}$$

$$\delta = \mathbf{G}'\zeta,$$

Then

$$\begin{aligned}
x_1 &= s_1(\mathbf{b_x'\Sigma_{x1}b_x})^{\frac{1}{2}} \\
&= s_1[\mathbf{d}'(s_1\mathbf{\Sigma_{x1}} + s_2\mathbf{\Sigma_{x2}})^{-1'}\mathbf{\Sigma_{x1}}(s_1\mathbf{\Sigma_{x1}} + s_2\mathbf{\Sigma_{x2}})^{-1}\mathbf{d}]^{\frac{1}{2}} \\
&= s_1[\mathbf{d}'(s_1\mathbf{\Sigma_{x1}'} + s_2\mathbf{\Sigma_{x2}'})^{-1}\mathbf{\Sigma_{x1}}(s_1\mathbf{\Sigma_{x1}} + s_2\mathbf{\Sigma_{x2}})^{-1}\mathbf{d}]^{\frac{1}{2}} \\
&= s_1[\zeta'\mathbf{GA}'((\mathbf{A\Sigma_1A}')'s_1 + (\mathbf{A\Sigma_2A}')'s_2)^{-1}\mathbf{A\Sigma_1A}'(\mathbf{A\Sigma_1A}'s_1 + \mathbf{A\Sigma_2A}'s_2)\mathbf{AG}'\zeta']^{\frac{1}{2}} \\
&= s_1[\zeta'\mathbf{GA}'(\mathbf{A\Sigma_1'A}'s_1 + \mathbf{A\Sigma_2'A}'s_2)^{-1}\mathbf{A\Sigma_1A}'(\mathbf{A\Sigma_1A}'s_1 + \mathbf{A\Sigma_2A}'s_2)\mathbf{AG}'\zeta']^{\frac{1}{2}} \\
&= s_1[\zeta'\mathbf{GA}'(\mathbf{AG}'\mathbf{\Lambda}\mathbf{GA}'s_1 + \mathbf{AG}'\mathbf{IGA}'s_2)^{-1}\mathbf{A\Sigma_1A}'(\mathbf{AG}'\mathbf{\Lambda}\mathbf{GA}'s_1 + \mathbf{AG}'\mathbf{IGA}'s_2)\mathbf{AG}'\zeta']^{\frac{1}{2}} \\
&= s_1[\zeta'\mathbf{GA}'(\mathbf{AG}'(\mathbf{\Lambda s_1} + \mathbf{Is_2})\mathbf{GA}')^{-1}\mathbf{AG}'\mathbf{\Lambda}\mathbf{GA}'(\mathbf{AG}'(\mathbf{\Lambda s_1} + \mathbf{Is_2})\mathbf{GA}')\mathbf{AG}'\zeta']^{\frac{1}{2}} \\
&= s_1[\zeta'(\mathbf{\Lambda s_1} + \mathbf{Is_2})^{-1}\mathbf{\Lambda}(\mathbf{\Lambda s_1} + \mathbf{Is_2})^{-1}\zeta]^{\frac{1}{2}} \\
&= s_1\left[\sum_{i=1}^{p}\frac{\zeta_{\mathbf{i}}^{\mathbf{2}}\lambda_{\mathbf{i}}}{(s_1\lambda_{\mathbf{i}} + s_2)^2}\right]^{\frac{1}{2}} \qquad (26)
\end{aligned}$$

$$
\begin{aligned}
x_2 &= s_2(\mathbf{b'_x \Sigma_{x2} b_x})^{\frac{1}{2}} \\
&= s_2[\mathbf{d}'(s_1\mathbf{\Sigma_{x1}} + s_2\mathbf{\Sigma_{x2}})^{-1'}\mathbf{\Sigma_{x2}}(s_1\mathbf{\Sigma_{x1}} + s_2\mathbf{\Sigma_{x2}})^{-1}\mathbf{d}]^{\frac{1}{2}} \\
&= s_2[\mathbf{d}'(s_1\mathbf{\Sigma'_{x1}} + s_2\mathbf{\Sigma'_{x2}})^{-1}\mathbf{\Sigma_{x2}}(s_1\mathbf{\Sigma_{x1}} + s_2\mathbf{\Sigma_{x2}})^{-1}\mathbf{d}]^{\frac{1}{2}} \\
&= s_2[\zeta'\mathbf{GA}'((\mathbf{A\Sigma_1A}')'s_1 + (\mathbf{A\Sigma_2A}')'s_2)^{-1}\mathbf{A\Sigma_2A}'(\mathbf{A\Sigma_1A}'s_1 + \mathbf{A\Sigma_2A}'s_2)\mathbf{AG}'\zeta']^{\frac{1}{2}} \\
&= s_2[\zeta'\mathbf{GA}'(\mathbf{A\Sigma'_1A}'s_1 + \mathbf{A\Sigma'_2A}'s_2)^{-1}\mathbf{A\Sigma_2A}'(\mathbf{A\Sigma_1A}'s_1 + \mathbf{A\Sigma_2A}'s_2)\mathbf{AG}'\zeta']^{\frac{1}{2}} \\
&= s_2[\zeta'\mathbf{GA}'(\mathbf{AG'\Lambda GA}'s_1 + \mathbf{AG'IGA}'s_2)^{-1}\mathbf{A\Sigma_2A}'(\mathbf{AG'\Lambda GA}'s_1 + \mathbf{AG'IGA}'s_2)\mathbf{AG}'\zeta']^{\frac{1}{2}} \\
&= s_2[\zeta'\mathbf{GA}'(\mathbf{AG}'(\mathbf{\Lambda s_1} + \mathbf{Is_2})\mathbf{GA}')^{-1}\mathbf{AG'IGA}'(\mathbf{AG}'(\mathbf{\Lambda s_1} + \mathbf{Is_2})\mathbf{GA}')\mathbf{AG}'\zeta']^{\frac{1}{2}} \\
&= s_2[\zeta'(\mathbf{\Lambda s_1} + \mathbf{Is_2})^{-1}(\mathbf{\Lambda s_1} + \mathbf{Is_2})^{-1}\zeta]^{\frac{1}{2}} \\
&= s_2\left[\sum_{i=1}^{p}\frac{\zeta_{\mathbf{i}}^{\mathbf{2}}}{(s_1\lambda_{\mathbf{i}} + s_2)^2}\right]^{\frac{1}{2}} \qquad (27)
\end{aligned}
$$

Therefore, by the same arguments in (16) and (17), x_1 is a monotonic increasing function of s_1 and s_2 is a monotonic decreasing function of $s_1 (0 \le s_1 \le 1)$.

5.1 Ether x_1 or x_2 is given

I use same matrices in section (4).

Suppose x_2 is given. If $x_2 = x_2^\star$, then $\mathbf{x_2} = s_2(\mathbf{b'_x\Sigma_{x2}b_x})^{\frac{1}{2}}$, where $\mathbf{b_x} = (s_1\mathbf{\Sigma_{x1}} + s_2\mathbf{\Sigma_{x2}})^{-1}\mathbf{d}$. Therefore

$$
\begin{aligned}
x_2^\star = (1-s_1)\left[\delta'\mathbf{A}'\{s_1\mathbf{A\Sigma_1A}' + (1-s_1)\mathbf{A\Sigma_2A}'\}^{-1}\mathbf{A\Sigma_2A}'\right. \\
\left.\{s_1\mathbf{A\Sigma_1A}' + (1-s_1)\mathbf{A\Sigma_2A}'\}^{-1}\mathbf{A}\delta\right]^{\frac{1}{2}} \qquad (28)
\end{aligned}
$$

Since $x_2^\star$ is known and x_2 is a decreasing function of x_1, I can easily approximate s_1 and compute $\mathbf{b_x} = (s_1\mathbf{\Sigma_{x1}} + s_2\mathbf{\Sigma_{x2}})^{-1}\mathbf{d}$.

Suppose $p(1|2)$ is 46.4%. In other words, $\Phi(x_2) = 1 - p(1|2)$ and $x_2^\star = 00.0913$. Since x_2 is decreasing function of s_1, I can approximate $s_1 = 0.93$. Therefore, the optimum vector $\mathbf{b_x}$ is

$$
\mathbf{b_x} = \begin{pmatrix} -0.685 & -0.405 & -0.696 & 0.0881 \end{pmatrix}'
$$

$$
x_1 = 0.6478 \qquad \text{by (24)}
$$

Therefore, the probability of misclassification $p(2|1)$ is

$$
p(2|1) = 1 - \Phi(x_1) = 1 - 0.85298 = 0.14702
$$

s_1	x_2	x_1	$p(1\|2)$	$p(2\|1)$	$p(1\|2) + p(2\|1)$
0.930	0.0913	1.0493	0.46363	0.14702	0.61065
0.780	0.2728	0.8400	0.39250	0.20045	0.59296
0.720	0.3407	0.7619	0.36666	0.22306	0.58972
0.650	0.4162	0.6745	0.33863	0.25000	0.58863
0.648	0.4189	0.6720	0.33764	0.25078	0.58842
0.647	0.4200	0.6708	0.33724	0.25117	**0.58841**
0.646	0.4210	0.6696	0.33686	0.25156	0.58842
0.640	0.4274	0.6623	0.33454	0.25389	0.58843
0.590	0.4796	0.6025	0.31576	0.27342	0.58918
0.580	0.4899	0.5907	0.31210	0.27736	0.58946
0.534	**0.5369**	**0.5369**	0.29567	0.29567	**0.59134**
0.500	0.5700	0.4990	0.28434	0.30889	0.59323
0.300	0.7575	0.2866	0.22438	0.38721	0.61158
0.200	0.8458	0.1878	0.19883	0.42552	0.62435
0.100	0.9319	0.0923	0.17569	0.46323	0.63892

5.2 Minimax procedure

Suppose $x_1 = x_2$. Since $x_1 > 0$ and $x_2 > 0$, $x_1^2 = x_2^2$.

$$\begin{aligned} 0 = x_1^2 - x_2^2 &= s_1^2 \mathbf{b_x'} \mathbf{\Sigma_{x1}} \mathbf{b_x} - (1-s_1)^2 \mathbf{b_x'} \mathbf{\Sigma_{x2}} \mathbf{b_x} \\ &= s_1^2 \mathbf{b_x'} \mathbf{A} \mathbf{\Sigma_1} \mathbf{A'} \mathbf{b_x} - (1-s_1)^2 \mathbf{b_x'} \mathbf{A} \mathbf{\Sigma_2} \mathbf{A'} \mathbf{b_x} \\ &= \mathbf{b_x'} \mathbf{A} [s_1^2 \mathbf{\Sigma_1} - (1-s_1)^2 \mathbf{\Sigma_2}] \mathbf{A'} \mathbf{b_x} \end{aligned}$$

I can guess a value of s_1 and solve the quadratic equation for $\mathbf{b_x}$. I get $s_1 \approx 0.534$.

$$x_1 = x_2 = 0.5369 \qquad \text{by (24) and (25)}$$

Therefore, the probability of misclassification $p(1|2)$ and $p(2|1)$ are

$$p(1|2) = p(2|1) = 1 - \Phi(x1) = 1 - 0.70433 = 0.29567$$

5.3 Case of a priori probabilities and cost function

If I are given a priori probabilities, p_1 and p_2, and the cost functions, $c(1|2)$ and $c(2|1)$, the probability of a misclassification is

$$p_1c(2|1)[1-\Phi(x_1)] + p_2c(1|2)[1-\Phi(x_2)]$$

The optimum solution can be found if I solve the following equation.

$$p_1c(2|1)\Phi(x_1)\frac{\partial x_1}{\partial t_1} + p_2c(1|2)\Phi(x_2)\frac{\partial x_2}{\partial t_1} = 0 \tag{29}$$

However, there is no direct way to solve the differential equation.

5.3.1 $\Sigma_{x1} = k\Sigma_{x2}$

$$\frac{p_1c(2|1)}{(\mathbf{b}'_\mathbf{x}\mathbf{\Sigma_{x1}}\mathbf{b_x})^{\frac{1}{2}}}\Phi(x_1) = \frac{p_2c(1|2)}{(\mathbf{b}'_\mathbf{x}\mathbf{\Sigma_{x2}}\mathbf{b_x})^{\frac{1}{2}}}\Phi(x_2) \tag{30}$$

$$\frac{p_1c(2|1)}{(\mathbf{b}'_\mathbf{x}\mathbf{\Sigma_{x1}}\mathbf{b_x})^{\frac{1}{2}}}\Phi(x_1) = \frac{p_2c(1|2)}{\frac{1}{\sqrt{k}}(\mathbf{b}'_\mathbf{x}\mathbf{\Sigma_{x2}}\mathbf{b_x})^{\frac{1}{2}}}\Phi(x_2)$$

$$(1-s_1-\sqrt{k}s_1)(1-s_1+\sqrt{k}s_1)\mathbf{b}'_\mathbf{x}\mathbf{\Sigma_{x1}}\mathbf{b_x} = k\ln k + 2k\ln\left(\frac{p_2c(1|2)}{p_1c(2|1)}\right)$$

$$(1-s_1-\sqrt{k}s_1)(1-s_1+\sqrt{k}s_1)\mathbf{b}'_\mathbf{x}\mathbf{A\Sigma_1A'}\mathbf{b_x} = k\ln k + 2k\ln\left(\frac{p_2c(1|2)}{p_1c(2|1)}\right) \tag{31}$$

From (4.3.1) and (26), I know that LHS of the equation is monotonic increasing or decreasing in s_1 depends on the sign of $(1-s_1-\sqrt{k}s_1)(1-s_1+\sqrt{k}s_1)$.

There are ECM's for the different k's below.

k	x_2	x_1	$p(1\|2)$	$p(2\|1)$	$0.5\,p(1\|2)+0.5\,p(2\|1)$
0.3	0.03	1.10	0.4878	0.1359	0.3119
0.5	0.21	0.86	0.4166	0.1958	0.3062
1.0	0.58	0.58	0.2819	0.2819	0.2819
1.5	0.81	0.50	0.2104	0.3096	0.2600
2.0	0.96	0.48	0.1688	0.3169	0.2428
3.0	1.16	0.49	0.1240	0.3129	0.2185
4.0	1.28	0.51	0.0995	0.3042	0.2018
5.0	1.38	0.54	0.0841	0.2952	0.1896

5.3.2 $\Sigma_{x1} = \Sigma_{x2}$

$$(1 - s_1 - \sqrt{k}s_1)(1 - s_1 + \sqrt{k}s_1)\mathbf{b'_x \Sigma_{x1} b_x} = k \ln k + 2k \ln\left(\frac{p_2 c(1|2)}{p_1 c(2|1)}\right)$$

$$(1 - 2s_1)\mathbf{b'_x \Sigma_{x1} b_x} = 0$$

Since $\mathbf{b'_x \Sigma_{x1} b_x}$ is positive definite matrix, $s_1 = \frac{1}{2}$. Therefore, replacing s_1 by $\frac{1}{2}$, the equation (12) gives the optimal discriminant

$$\begin{aligned}\mathbf{d} &= (s_1\mathbf{\Sigma_{x1}} + s_2\mathbf{\Sigma_{x2}})\mathbf{b_x}\\ &= (\frac{1}{2}\mathbf{\Sigma_{x1}} + \frac{1}{2}\mathbf{\Sigma_{x2}})\mathbf{b_x}\\ &= \frac{1}{2}(\mathbf{\Sigma_{x1}} + \mathbf{\Sigma_{x1}})\\ &= \mathbf{\Sigma_{x1} b_x}\end{aligned}$$

$$\mathbf{b_x} = \mathbf{\Sigma_{x1}^{-1} d} \tag{32}$$

The optimal vector $\mathbf{b_x}$ is same as the linear discriminant $\mathbf{l_x} = \mathbf{\Sigma_x^{-1}}\mu_\mathbf{d}$ (Arya et al., 2000).

6 Concluding Remarks

The educational purpose of this paper is providing accounting students with better understanding of the nature of accounting procedure. The preparation of financial statements is nothing but a linear process of accounting information aggregation. It is inevitable to lose information through the preparation process of financial statements (i.e. aggregation). The aggregation process provides benefits as well. One of the aggregation gains is related to the bounded rationality (Arya et al., 2000). More information may not be always optimal since the interpretation for overloaded information causes costs including time and money. Hence, many investors refer to audited financial statements for their decision making. Another potential benefit is related to the measurement errors. Measurement error in specific items may be canceled out through the aggregation process (Grunfeld and Griliches, 1960; Lim and Sunder, 1991; Datar and Gupta, 1994).

Accounting students can acquire better understanding of the mathematical implications of accounting procedure from this exercise. Although this study provides general models for accounting discriminant analysis, next stage of development will likely extend this study in several directions. One case is that on average managers show equal mean matrix and unequal variance matrices in their transactions. To begin with, it will set up an agency model for the earnings management. I conjecture that investors (i.e. principal) can be better off from annual or quarterly reports (i.e. aggregated information) in the presence of volatility in the reported accounts of companies. There are two companies, managing earnings and non-managing earnings. Although two companies show same reported numbers in ending balance, investors can discriminated one from the other by checking the variance of accounts. In this regard, it would be worthwhile for future research to look into variability of accounting information over a longer interval.

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