Aggregation, Uncertainty, and Discriminant Analysis

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Abstract: An important role of accountants is to provide the financial summary for the economic activities of companies over the period. The preparation of financial statements is nothing but a linear aggregation process of accounting information. The numbers summarized in financial statements balance (i.e. aggregated financial information) computed from the aggregation process of a myriad of day-to-day transactions (i.e. disaggregated financial information) of companies. The consumers of accounting information utilize aggregated or disaggregated accounting information in their decision making. Discriminant analysis is a pervasive field of which accounting information is used to separate distinct sets of entities. This paper provide practices of examining the effects of aggregation by examining the impacts of aggregation on the discriminating between two entities. In addition, the students who take financial accounting courses can be benefited by understanding the accounting nature of linear procedures.

Key words: Aggregation, Discriminant Analysis, Unequal variance

JEL classification: M40, M41, M49

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1Preliminary version
1 Introduction

An important role of accountants is to provide the financial summary to decision makers pertaining to the economic activities of companies over the period. The preparation of financial statements is nothing but a linear process of accounting information aggregation. The numbers summarized in financial statements balance (i.e. aggregated financial information) computed from the aggregation process of a myriad of day-to-day transactions (i.e. disaggregated financial information) of the companies. Accounting system collects and processes financial information of companies, and summaries and reports comprehensive information in relatively few line items. For example, financial accounting collects and processes information regarding daily transactions between the company and suppliers. Accountants aggregates those transactions and report in accounting balances including account payables, cash, or inventory. Managerial accounting aggregates various product costs into cost of good sold.

Discriminant analysis is applied to separate distinct sets of entities. In particular, this study investigate the relationship between discrimination and accounting aggregation procedure. Various applications can follow the discriminant analysis. For example, an auditor can use discriminant analysis in evaluating financial statements of audit clients (Koh and Killough, 1990). Bankruptcy prediction is another pervasive theme in applying the discriminant analysis to business (Altman, 1968; Balcaen and Ooghe, 2006). Banks can determine whether a firm should be classified as high credit risk or low credit risk using financial statement (i.e. aggregated accounting information). In doing so, they can also estimate the costs of misclassifying the entity. Explicit costs would be attached to the misclassifying an entity. A bank approves a loan to a firm by incorrectly classifying the company as low credit risk increases the likelihood that it would suffer from the potential loss due to the default of the company. On the other hand, a bank rejects a loan to a firm by incorrectly classifying the company as high credit is subject to the potential loss of profit opportunity. Therefore, accurate discrimination process is of substantial importance to various stakeholders.

An optimal decision rule for the discrimination is to minimize the average or expected cost of misclassification (ECM). Aforementioned example shows two types of errors are
associated with ECM. Classifying a firm as not likely to default when it does default is *Type I error*. On the other hand, Classifying a firm as likely to default when it does not default is *Type II error*. It is assumed that the decision makers adopt the classification scheme evaluated in terms of ECM.

Arya et al. (2000) examines the costs and benefits of aggregating financial information in the discriminating between two entities. They analyze the model in the context that the two entities fundamentally differ in their business activities. That is, the fundamental assumption of the previous research is that two entities have a common covariance matrix in transaction but have different mean value of their transaction matrices. While the equal covariance case uses linear discriminant function for the discrimination, unequal covariance case utilizes quadratic function. One purpose of this paper is providing accounting students with better understanding of accounting aggregation through the linear procedure. In the next section, the general discriminant model with unequal covariance matrices will be derived.

### 2 Basic Model of Classification

I assume that all parameters are known and I will follow linear procedures. Let \( \pi_1 \) and \( \pi_2 \) be \( N(\mu_1, \Sigma_1) \) and \( N(\mu_2, \Sigma_2) \) with \( \mu_1 \neq \mu_2 \) and \( \Sigma_1 \neq \Sigma_2 \), since the case \( \Sigma_1 = \Sigma_2 \) has been treated in the seminar. I assume that \( \Sigma_1 \) and \( \Sigma_2 \) are nonsingular.

**Notation 1** Let \( b \neq 0 \) be a vector of \( p \) components and \( c \) be a scalar.

**Notation 2** \( \Phi(z) = \int_{-\infty}^{z} (2\pi)^{-\frac{1}{2}} e^{-\frac{1}{2}t^2} dt \)

An observation \( x \) is classified as from \( \pi_1 \) if \( b'x \leq c \) and as from \( \pi_2 \) if \( b'x > c \). \( b'x \) is a univariate normal distribution.

The mean is

\[
E(b'x) = b'\mu_i, \quad i = 1, 2
\]  

The variance is
\[
E(b'x - b'_\mu)^2 = E(b'(x - \mu_1)(x - \mu_1)'b) \\
= b'\Sigma_ib, \quad i = 1, 2 \tag{2}
\]

3 Benchmark Solutions

\[
P(2 \mid 1) = \Pr\{b'x > c \mid \pi_1\} \\
= \Pr\left\{\frac{b'x - b'_\mu_1}{(b'\Sigma_1b)^{\frac{1}{2}}} > \frac{c - b'_\mu_1}{(b'\Sigma_1b)^{\frac{1}{2}}} \mid \pi_1\right\} \\
= 1 - \Phi\left(\frac{c - b'_\mu_1}{(b'\Sigma_1b)^{\frac{1}{2}}}\right) \tag{3}
\]

\[
P(1 \mid 2) = \Pr\{b'x \leq c \mid \pi_2\} \\
= \Pr\left\{\frac{b'x - b'_\mu_2}{(b'\Sigma_2b)^{\frac{1}{2}}} \leq \frac{c - b'_\mu_2}{(b'\Sigma_2b)^{\frac{1}{2}}} \mid \pi_2\right\} \\
= 1 - \Phi\left(\frac{b'_\mu_2 - c}{(b'\Sigma_2b)^{\frac{1}{2}}}\right) \tag{4}
\]

I want to minimize these two probabilities. In other words, I desire to maximize the following arguments

\[
y_1 = \frac{c - b'_\mu_1}{(b'\Sigma_1b)^{\frac{1}{2}}} \tag{5}
\]

\[
y_2 = \frac{b'_\mu_2 - c}{(b'\Sigma_2b)^{\frac{1}{2}}} \tag{6}
\]

From the equation (6),

\[
c = b'_\mu_2 - y_2(b'\Sigma_2b)^{\frac{1}{2}} \tag{7}
\]

Then

\[
y_1 = \frac{b'_\mu_2 - b'_\mu_1 - y_2(b'\Sigma_2b)^{\frac{1}{2}}}{(b'\Sigma_1b)^{\frac{1}{2}}} \tag{8}
\]

Let \(\delta = \mu_2 - \mu_1\)

\[
y_1 = \frac{b'\delta - y_2(b'\Sigma_2b)^{\frac{1}{2}}}{(b'\Sigma_1b)^{\frac{1}{2}}} \tag{8}
\]
I differentiate \( y_1 \) with respect to \( b \) to maximize \( y_1 \) given \( y_2 \).

\[
\frac{\partial y_1}{\partial b} = \left( \delta - \frac{1}{2} y_2 (b' \Sigma_2 b)^{-\frac{1}{2}} 2 \Sigma_2 b \right) \left( b' \Sigma_1 b \right)^{-\frac{1}{2}} \\
- \frac{1}{2} (b' \Sigma_1 b)^{-\frac{3}{2}} (b' \delta - y_2 (b' \Sigma_2 b)^{\frac{1}{2}}) 2 \Sigma_1 b
\]

\[
= \delta (b' \Sigma_1 b)^{-\frac{3}{2}} - y_2 (b' \Sigma_2 b)^{-\frac{1}{2}} 2 \Sigma_2 b (b' \Sigma_1 b)^{-\frac{1}{2}} \\
- b' \delta (b' \Sigma_2 b)^{-\frac{3}{2}} \Sigma_1 b + y_2 (b' \Sigma_2 b)^{\frac{1}{2}} (b' \Sigma_1 b)^{-\frac{3}{2}} \Sigma_1 b
\]

\[
= (b' \Sigma_1 b)^{-\frac{3}{2}} \left[ \delta - y_2 (b' \Sigma_2 b)^{-\frac{1}{2}} 2 \Sigma_2 b - b' \delta (b' \Sigma_1 b)^{-1} \Sigma_1 b \\
+ y_2 (b' \Sigma_2 b)^{\frac{1}{2}} (b' \Sigma_1 b)^{-1} \Sigma_1 b \right] = 0
\]

Since \((b' \Sigma_1 b)^{-\frac{3}{2}}\) is positive definite

\[
\left[ \delta - y_2 (b' \Sigma_2 b)^{-\frac{1}{2}} 2 \Sigma_2 b - b' \delta (b' \Sigma_1 b)^{-1} \Sigma_1 b + y_2 (b' \Sigma_2 b)^{\frac{1}{2}} (b' \Sigma_1 b)^{-1} \Sigma_1 b \right] = 0
\]

\[
\delta = y_2 (b' \Sigma_2 b)^{-\frac{1}{2}} 2 \Sigma_2 b - b' \delta (b' \Sigma_1 b)^{-1} \Sigma_1 b + y_2 (b' \Sigma_2 b)^{\frac{1}{2}} (b' \Sigma_1 b)^{-1} \Sigma_1 b
\]

\[
= \left( \frac{y_2}{(b' \Sigma_2 b)^{\frac{1}{2}}} \right) \Sigma_2 + \left( \frac{b' \delta - y_2 (b' \Sigma_2 b)^{\frac{1}{2}}}{b' \Sigma_1 b} \right) \Sigma_1
\]

Let

\[
t_1 = \frac{b' \delta - y_2 (b' \Sigma_2 b)^{\frac{1}{2}}}{b' \Sigma_1 b} \quad (10)
\]

\[
t_2 = \frac{y_2}{(b' \Sigma_2 b)^{\frac{1}{2}}} \quad (11)
\]

Then

\[
\delta = (t_1 \Sigma_1 + t_2 \Sigma_2) b
\]

From (5) and (11)

\[
c = b' \mu_2 - y_2 (b' \Sigma_2 b)^{\frac{1}{2}}
\]

\[
= b' \mu_2 - t_2 b' \Sigma_2 b
\]

From (8) and (10)

\[
y_1 = \frac{b' \delta - y_2 (b' \Sigma_2 b)^{\frac{1}{2}}}{(b' \Sigma_1 b)^{\frac{1}{2}}}
\]

\[
= t_1 (b' \Sigma_1 b)^{\frac{1}{2}}
\]

(14)
From (11)

\[ y_2 = t_2(b'\Sigma_2 b)^{\frac{3}{2}} \]  

(15)

Note that the right hand sides of (14) and (15) are homogeneous of degree 0 in \( t_1 \) and \( t_2 \). In other words, if I plug (12) in (14) and (15), I always get same values in \( y_1 \) and \( y_2 \) regardless of \( t_1 \) and \( t_2 \). Therefore it is convenient if I normalize \( t_1 \) and \( t_2 \) such that

\[ t_1 + t_2 = 1 \]

If I can show that \( y_1 \) is a monotonic increasing function of \( t_1 \) and \( y_2 \) is a monotonic decreasing function of \( t_1 (0 \leq t_1 \leq 1) \), I can calculate optimum \( b \) (I will discuss this later).

Since \( \Sigma_1 \) and \( \Sigma_2 \) are positive definite matrices, I can use Cholesky decomposition. In other words, there exists a matrix \( R \) with independent columns. For the convenience, I can transform the covariance matrices to the following form

\[ \Sigma_2 = R'R, \]

\[ \Sigma_1 = R'\Lambda R = R' \begin{pmatrix} \lambda_1 & 0 & \ldots & 0 \\ 0 & \lambda_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \lambda_p \end{pmatrix} R, \quad \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p > 0 \]

\[ \delta = R'\theta, \]

From (12)

\[ b = (t_1 \Sigma_1 + t_2 \Sigma_2)^{-1}\delta \]

\[ b' = \delta'(t_1 \Sigma_1 + t_2 \Sigma_2)^{-1}' \]
Then

\[ y_1 = t_1 (b' \Sigma_1 b) ^{\frac{1}{2}} \]

\[ = t_1 [\sigma^2 (t_1 \Sigma_1 + t_2 \Sigma_2)^{-1} \Sigma_1 (t_1 \Sigma_1 + t_2 \Sigma_2)^{-1} \delta] ^{\frac{1}{2}} \]

\[ = t_1 [\sigma^2 (t_1 \Sigma_1' + (t_2 \Sigma_2')^{-1} \Sigma_1 (t_1 \Sigma_1 + t_2 \Sigma_2)^{-1} R' \theta] ^{\frac{1}{2}} \]

\[ = t_1 [\sigma^2 (t_1 \Sigma_1' t_1 + \Sigma_2' t_2)^{-1} \Sigma_1 (t_1 \Sigma_1 + t_2 \Sigma_2)^{-1} R' \theta] ^{\frac{1}{2}} \]

\[ = t_1 [\sigma^2 (R' \Lambda_t t_1 + R' \Theta t_2)^{-1} R' \Lambda t_1 R' \Lambda t_2 R' \Theta t_2)^{-1} R' \theta] ^{\frac{1}{2}} \]

\[ = t_1 [\sigma^2 (R' \Lambda t_1 + t_2 I)^{-1} R' \Lambda t_1 R' \Lambda t_2 I R' \Theta t_2)^{-1} R' \theta] ^{\frac{1}{2}} \]

\[ = t_1 [\sigma^2 (\Lambda t_1 + t_2 I)^{-1} \Lambda (\Lambda t_1 + t_2 I)^{-1} \theta] ^{\frac{1}{2}} \]

\[ = t_1 \left[ \sum_{i=1}^{p} \frac{\theta_i^2 \lambda_i}{(t_1 \lambda_i + t_2)^2} \right] ^{\frac{1}{2}} \] (16)

\[ y_2 = t_2 (b' \Sigma_2 b) ^{\frac{1}{2}} \]

\[ = t_2 [\sigma^2 (t_1 \Sigma_1 + t_2 \Sigma_2)^{-1} \Sigma_2 (t_1 \Sigma_1 + t_2 \Sigma_2)^{-1} \delta] ^{\frac{1}{2}} \]

\[ = t_2 [\sigma^2 (t_1 \Sigma_1' + (t_2 \Sigma_2')^{-1} \Sigma_2 (t_1 \Sigma_1 + t_2 \Sigma_2)^{-1} R' \theta] ^{\frac{1}{2}} \]

\[ = t_2 [\sigma^2 (t_1 \Sigma_1' t_1 + \Sigma_2' t_2)^{-1} \Sigma_2 (t_1 \Sigma_1 + t_2 \Sigma_2)^{-1} R' \theta] ^{\frac{1}{2}} \]

\[ = t_2 [\sigma^2 (R' \Lambda t_1 + R' \Theta t_2)^{-1} R' \Theta t_2)^{-1} R' \theta] ^{\frac{1}{2}} \]

\[ = t_2 [\sigma^2 (R' (\Lambda t_1 + t_2 I) R)^{-1} R' (\Lambda t_1 + t_2 I) R)^{-1} R' \theta] ^{\frac{1}{2}} \]

\[ = t_2 [\sigma^2 (\Lambda t_1 + t_2 I)^{-1} (t_1 \Lambda + t_2 I)^{-1} R' \theta] ^{\frac{1}{2}} \]

\[ = t_2 \left[ \sum_{i=1}^{p} \frac{\theta_i^2 \lambda_i}{(t_1 \lambda_i + t_2)^2} \right] ^{\frac{1}{2}} \] (17)

Since \( y_1 > 0 \) and \( y_2 > 0 \), it is convenient to take the derivative of \( y_1^2 \) instead of \( y_1 \) where \( t_2 = 1 - t_1 \).

\[ y_1^2 = t_1^2 \sum_{i=1}^{p} \frac{\theta_i^2 \lambda_i}{(t_1 \lambda_i + t_2)^2} \]

\[ = t_1^2 \frac{\theta_1^2 \lambda_1}{(t_1 \lambda_1 + t_1)^2} + \ldots \]
\[
y_2^2 = t_2^2 \sum_{i=1}^{p} \frac{\theta_i^2}{(t_1 \lambda_i + t_2)^2} \\
= t_2^2 \frac{\theta_1^2}{(t_1 \lambda_1 + 1 - t_1)^2} + \cdots
\]

\[
\frac{\partial y_1^2}{\partial t_1} = \frac{2 t_1 \theta_1^2 \lambda_1 (t_1 \lambda_1 + 1 - t_1)^2 - 2(t_1 \lambda_1 + 1 - t_1)(\lambda_1 - 1)t_1^2 \theta_1^2 \lambda_1}{(t_1 \lambda_1 + 1 - t_1)^4} + \cdots
\]

\[
= \frac{2 t_1 \theta_1^2 \lambda_1}{(t_1 \lambda_1 + 1 - t_1)^3} + \cdots
\]

\[
= 2 t_1 \sum_{i=1}^{p} \frac{\theta_i^2 \lambda_i}{(t_1 \lambda_i + 1 - t_1)^3} > 0
\]

\[
\frac{\partial y_2^2}{\partial t_1} = \frac{-2(1 - t_1) \theta_1^2 (t_1 \lambda_1 + 1 - t_1)^2 - 2(t_1 \lambda_1 + 1 - t_1)(\lambda_1 - 1)(1 - t_1)^2 \theta_1^2}{(t_1 \lambda_1 + 1 - t_1)^4} + \cdots
\]

\[
= \frac{-2(1 - t_1) \theta_1^2 \lambda_1}{(t_1 \lambda_1 + 1 - t_1)^3} + \cdots
\]

\[
= -2(1 - t_1) \sum_{i=1}^{p} \frac{\theta_i^2 \lambda_i}{(t_1 \lambda_i + 1 - t_1)^3} < 0
\]

Therefore, \( y_1 \) is a monotonic increasing function of \( t_1 \) and \( y_2 \) is a monotonic decreasing function of \( t_1 (0 \leq t_1 \leq 1) \).

### 4 Use of Transaction Matrices

The double entry transformation matrix \( A \) and the six transactions are

\[
\begin{align*}
    t_1 & : \text{collections of accounts receivable} \\
    t_2 & : \text{cash purchase of inventory} \\
    t_3 & : \text{credit sales} \\
    t_4 & : \text{cost of goods sold recognized} \\
    t_5 & : \text{cash sales} \\
    t_6 & : \text{cash expenses}
\end{align*}
\]
\[ A = \begin{pmatrix}
1 & -1 & 0 & 0 & 1 & -1 \\
-1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & -1 \\
0 & 0 & 0 & 1 & 0 & 1 \\
\end{pmatrix} \]

Cash

A/R

Inventory

Sales

Expenses

The covariance matrices are

\[ \Sigma_1 = \begin{pmatrix}
0.3 & 0.1 & 0.2 & 0 & 0 & 0 \\
0 & 1.4 & 0 & 0.4 & 0 & 0 \\
0.1 & 0 & 1.2 & 0 & 0 & 0.3 \\
0.1 & 0 & 0 & 0.9 & 0 & 0 \\
0 & 0.3 & 0 & 0.3 & 1.0 & 0 \\
0.2 & 0 & 0.5 & 0 & 0 & 4 \\
\end{pmatrix} \]

\[ \Sigma_2 = \begin{pmatrix}
0.4 & 0.2 & 0.3 & 0 & 0 & 0 \\
0 & 1.7 & 0 & 0.4 & 0 & 0 \\
0 & 0 & 1.7 & 0 & 0.4 & 0 \\
0.1 & 0 & 0 & 1.3 & 0 & 0 \\
0.2 & 0.3 & 0 & 0 & 1.4 & 0 \\
0.6 & 0.5 & 0 & 0 & 0 & 0.8 \\
\end{pmatrix} \]

Suppose the mean transaction matrices are

\[ \mu_1 = \begin{pmatrix} 4.25 & 5.25 & 4 & 5 & 1.5 \end{pmatrix}' \]

\[ \mu_2 = \begin{pmatrix} 4.25 & 5.25 & 5 & 6 & 1.5 \end{pmatrix}' \]

If \( t_1 \) and \( t_2 \) are given, I can calculate optimal \( b \) by the equation (12). Then I can compute \( c \) by the equation (7). However, \( t_1 \) and \( t_2 \) are rarely known. So I should restrict our case to the following way.
4.1 Ether $y_1$ or $y_2$ is given

Suppose that $y_2$ is given. If $y_2 = y_2^*$, then $y_2^* = t_2(b'\Sigma_2b)^{\frac{1}{2}}$, where $b = (t_1\Sigma_1 + t_2\Sigma_2)^{-1}\delta$. Therefore

$$y_2^* = (1 - t_1)\left[\delta'(t_1\Sigma_1 + (1 - t_1)\Sigma_2)^{-1}\Sigma_2(t_1\Sigma_1 + (1 - t_1)\Sigma_2)^{-1}\delta\right]^{\frac{1}{2}} \quad (18)$$

Since $y_2^*$ is known and $y_2$ is a decreasing function of $t_1$, I can easily approximate $t_1$ by trial and error ($0 < t_1 \leq 1$).

Now I can compute $b = \{t_1\Sigma_1 + (1 - t_1)\Sigma_2\}^{-1}\delta$. This is minimizing misclassification error.

Suppose $p(1|2)$ is known to be 46%. In other words, $\Phi(y_2) = 1 - p(1|2)$ and $y_2^* = 0.1$. Since $y_2$ is decreasing function of $t_1$, I can try other values of $t_1$ and insert in the equation (18) until I get the value $y_2 \approx y_2^*$. I can compute $t_1 \approx 0.93$ by trial and error. Therefore, the optimum vector $b$ is computed by (12).

$$b = \begin{pmatrix} 0.157 & -0.258 & -0.0796 & 0.961 & 0.772 & 0.00543 \end{pmatrix}^T$$

$y_1 = 0.1$ \hspace{1cm} by (5)

Therefore, the probability of misclassification $p(2|1)$ is

$$p(2|1) = 1 - \Phi(y_1) = 1 - 0.89435 = 0.10565$$
4.2 Minimax procedure

Suppose $y_1 = y_2$. This equality is same as $y_1^2 = y_2^2$ because $y_1 > 0$ and $y_2 > 0$.

\[
0 = y_1^2 - y_2^2 = t_1^2 b' \Sigma_1 b - (1 - t_1)^2 b' \Sigma_2 b
\
= b' [t_1^2 \Sigma_1 - (1 - t_1)^2 \Sigma_2] b
\]

In this case, I can guess a value of $t_1$ and solve the quadratic equation for $b$. I get $t_1 \approx 0.5275$.

\[
y_1 = y_2 = 0.6448 \quad \text{by (5) and (6)}
\]

Therefore, the probability of misclassification $p(1|2)$ and $p(2|1)$ are

\[
p(1|2) = p(2|1) = 1 - \Phi(y_1) = 1 - 0.74047 = 0.25953
\]
4.3 Case of a priori probabilities and cost function

If I am given a priori probabilities, \( p_1 \) and \( p_2 \), and the cost functions, \( c(1|2) \) and \( c(2|1) \), the probability of a misclassification is

\[
p_1 c(2|1) [1 - \Phi(y_1)] + p_2 c(1|2) [1 - \Phi(y_2)]
\]

If I take derivative of the equation above

\[
p_1 c(2|1) \Phi(y_1) \frac{\partial y_1}{\partial t_1} + p_2 c(1|2) \Phi(y_2) \frac{\partial y_2}{\partial t_1} = 0 \tag{19}
\]

There is no easy solution to the differential equation (19).

4.3.1 \( \Sigma_1 = k \Sigma_2 \)

\[
\frac{\partial y_2}{\partial t_1} = -\frac{(b' \Sigma_2 b)^{-\frac{1}{2}}}{(b' \Sigma_1 b)^{-\frac{1}{2}}} \quad \text{by the Envelop theorems 1994}
\]

Therefore, the equation (19) can be expressed as

\[
\frac{p_1 c(2|1)}{(b' \Sigma_1 b)^{\frac{1}{2}}} \Phi(y_1) = \frac{p_2 c(1|2)}{(b' \Sigma_2 b)^{\frac{1}{2}}} \Phi(y_2) \tag{20}
\]

\[
\phi(y_1) = \frac{p_2 c(1|2)}{p_1 c(2|1)} \sqrt{k} \Phi(y_2)
\]

\[
(2\pi)^{-\frac{1}{2}} e^{-\frac{1}{2} y_1^2} = \frac{p_2 c(1|2)}{p_1 c(2|1)} \sqrt{k} (2\pi)^{-\frac{1}{2}} e^{-\frac{1}{2} y_2^2}
\]

\[
e^{-\frac{1}{2} \left( t_1^2 (b' \Sigma_1 b) - (1-t_1)^2 (b' \Sigma_2 b) \right)} = \frac{p_2 c(1|2)}{p_1 c(2|1)} \sqrt{k}
\]

\[
-\frac{1}{2} t_1^2 (b' \Sigma_1 b) - \frac{(1-t_1)^2}{k} (b' \Sigma_1 b) = \ln \frac{p_2 c(1|2)}{p_1 c(2|1)} + \frac{1}{2} \ln k
\]

\[
( -kt_1^2 + (1-t_1)^2 ) b' \Sigma_1 b = k \ln k + 2k \ln \left( \frac{p_2 c(1|2)}{p_1 c(2|1)} \right)
\]

\[
( (1-t_1)^2 - \sqrt{k} t_1^2 ) b' \Sigma_1 b = k \ln k + 2k \ln \left( \frac{p_2 c(1|2)}{p_1 c(2|1)} \right)
\]

\[
(1-t_1 - \sqrt{k} t_1 ) (1-t_1 + \sqrt{k} t_1 ) b' \Sigma_1 b = k \ln k + 2k \ln \left( \frac{p_2 c(1|2)}{p_1 c(2|1)} \right) \tag{21}
\]

As I proved in (16) and (17), \( b' \Sigma_1 b \) is a monotonic increasing function in \( t_1 \). Therefore, whether LHS of the equation is monotonic increasing or decreasing in \( t_1 \) depends on the
sign of \((1-t_1-\sqrt{kt_1})(1-t_1+\sqrt{kt_1})b'\Sigma_1 b\). Since \(0 \leq t_1 \leq 1\), I can consider the following cases.

<table>
<thead>
<tr>
<th>(k)</th>
<th>(\frac{1}{1+\sqrt{k}} &lt; t_1 &lt; 1)</th>
<th>Monotonic Decreasing</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k &lt; 1)</td>
<td>(0 \leq t_1 \leq \frac{1}{1+\sqrt{k}})</td>
<td>Monotonic Increasing</td>
</tr>
<tr>
<td>(k \geq 1)</td>
<td>(0 &lt; t_1 &lt; \frac{1}{1+\sqrt{k}})</td>
<td>Monotonic Decreasing</td>
</tr>
<tr>
<td>(k \geq 1)</td>
<td>(\frac{1}{1+\sqrt{k}} \leq t_1 \leq 1)</td>
<td>Monotonic Increasing</td>
</tr>
</tbody>
</table>

Suppose \(\Sigma_1 = 2\Sigma_2\), \(p_1 = p_2 = 0.5\), and \(c(1|2) = c(2|1) = 1\). RHS of the equation (21) becomes \(k \ln k = 1.386\). Since LHS is monotonic decreasing in \(t_1\), I can easily compute \(t_1\) by trial and error. In this case, \(t_1 = 0.2986\), \(y_1 = 0.63\) and \(y_2 = 1.04\). As a result, the total cost of misclassification is

\[
0.5(1 - \Phi y_1) + 0.5(1 - \Phi y_2) = 0.5(0.1492 + 0.2643) = 0.2063
\]

There are ECM’s for the different \(k\)’s below.

| \(k\) | \(y_2\) | \(y_1\) | \(p(1|2)\) | \(p(2|1)\) | \(0.5 p(1|2) + 0.5 p(2|1)\) |
|-------|-------|-------|---------|---------|-----------------|
| 0.3   | 1.12  | 0.14  | 0.1314  | 0.4443  | 0.2878          |
| 0.5   | 0.90  | 0.32  | 0.1841  | 0.3745  | 0.2793          |
| 1.0   | 0.68  | 0.68  | 0.2483  | 0.2483  | 0.2483          |
| 1.5   | 0.52  | 0.89  | 0.3015  | 0.1867  | 0.2441          |
| 2.0   | 0.63  | 1.04  | 0.2643  | 0.1492  | 0.2068          |
| 3.0   | 0.65  | 1.23  | 0.2578  | 0.1093  | 0.1836          |
| 4.0   | 0.68  | 1.36  | 0.2483  | 0.0869  | 0.1676          |
| 5.0   | 0.71  | 1.45  | 0.2389  | 0.0735  | 0.1562          |

### 4.3.2 \(\Sigma_1 = \Sigma_2\)

I want to show the result of the linear procedure is consistent with the analysis described in the paper (Arya et al., 2000).

\[
(1-t_1-\sqrt{kt_1})(1-t_1+\sqrt{kt_1})b'\Sigma_1 b = k \ln k + 2k \ln \left(\frac{p_2 c(1|2)}{p_1 c(2|1)}\right)
\]

\[
(1-2t_1)b'\Sigma_1 b = 0
\]
Since \( b' \Sigma_1 b \) is positive definite matrix, \( t_1 = \frac{1}{2} \). Therefore, if I plug \( t_1 \) in the equation (12),

\[
\delta = (t_1 \Sigma_1 + t_2 \Sigma_2)b \\
= (\frac{1}{2} \Sigma_1 + \frac{1}{2} \Sigma_2)b \\
= \frac{1}{2} (\Sigma_1 + \Sigma_1) \\
= \Sigma_1 b \\
\]

\[
b = \Sigma^{-1} \delta
\]

This vector is identical to the linear discriminant \( l_y = \Sigma_y^{-1} \eta_d \) (Arya et al., 2000).

5 Use of Balance Matrices

It is inevitable to lose information during the aggregation process since balance vector \( x \) is shorter than transaction vector \( y \) (i.e. \( m \leq n \)).

If I use the financial statement vector \( x \) instead of the transaction vector \( y \),

\[
Ay = x
\]

\[
A = \begin{pmatrix} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}
\]

\[
\Sigma_{x1} = A \Sigma_1 A' \\
\Sigma_{x2} = A \Sigma_2 A' \\
d = A \delta
\]

\[
b_x = (s_1 \Sigma_{x1} + s_2 \Sigma_{x2})^{-1} d 
\]

\[
x_1 = s_1 (b_x' \Sigma_{x1} b_x)^{\frac{1}{2}} \
\]

\[
x_2 = s_2 (b_x' \Sigma_{x2} b_x)^{\frac{1}{2}}
\]
Now I want to show that $x_1$ is a monotonic increasing function of $s_1$ and $x_2$ is a monotonic decreasing function of $s_1(0 \leq t_1 \leq 1)$. Since $\Sigma_1$ and $\Sigma_2$ are positive definite matrices, there exists a rectangular matrix $G_{4 \times 6}$. For the convenience, I can transform the covariance matrices to the following form

$$\Sigma_2 = G'G,$$

$$\Sigma_1 = G'\Lambda G = G'
\begin{pmatrix}
\lambda_1 & 0 & 0 & 0 \\
0 & \lambda_2 & 0 & 0 \\
0 & 0 & \lambda_3 & 0 \\
0 & 0 & 0 & \lambda_4
\end{pmatrix}G$$

$$\delta = G'\zeta,$$

Then

$$x_1 = s_1(b'_x \Sigma_{x1} b_x)^{\frac{1}{2}}$$

$$= s_1[d'(s_1 \Sigma_{x1} + s_2 \Sigma_{x2})^{-1} \Sigma_{x1}(s_1 \Sigma_{x1} + s_2 \Sigma_{x2})^{-1}d]^{\frac{1}{2}}$$

$$= s_1[d'(s_1 \Sigma'_{x1} + s_2 \Sigma'_{x2})^{-1} \Sigma_{x1}(s_1 \Sigma_{x1} + s_2 \Sigma_{x2})^{-1}d]^{\frac{1}{2}}$$

$$= s_1[\zeta'G'((A \Sigma_1 A')s_1 + (A \Sigma_2 A')s_2)^{-1} A\Sigma_1 A'(A \Sigma_1 A's_1 + A \Sigma_2 A's_2)AG'\zeta']^{\frac{1}{2}}$$

$$= s_1[\zeta'G'(A \Sigma_1 A's_1 + A \Sigma_2 A's_2)^{-1} A\Sigma_1 A'(A \Sigma_1 A's_1 + A \Sigma_2 A's_2)AG'\zeta']^{\frac{1}{2}}$$

$$= s_1[\zeta'G'(AG'AG's_1 + AG'IGA's_2)^{-1} A\Sigma_1 A'(AG'AG's_1 + AG'IGA's_2)AG'\zeta']^{\frac{1}{2}}$$

$$= s_1[\zeta'(A\Sigma_1 + I_{s_2})GA')^{-1} AG'AG'(A\Sigma_1 + I_{s_2})GA')AG'\zeta']^{\frac{1}{2}}$$

$$= s_1[\zeta'(A\Sigma_1 + I_{s_2})^{-1} A(A\Sigma_1 + I_{s_2})^{-1}\zeta']^{\frac{1}{2}}$$

$$= s_1 \left[ \sum_{i=1}^{p} \frac{\zeta_i^2 \lambda_i}{(s_1 \lambda_i + s_2)^2} \right]^{\frac{1}{2}} \tag{26}$$
\[x_2 = s_2(b'_x \Sigma x_2 b_x)^{\frac{1}{2}}\]
\[= s_2[d'(s_1 \Sigma x_1 + s_2 \Sigma x_2)^{-1} \Sigma x_2(s_1 \Sigma x_1 + s_2 \Sigma x_2)^{-1} d]^{\frac{1}{2}}\]
\[= s_2[d'(s_1 \Sigma' x_1 + s_2 \Sigma' x_2)^{-1} \Sigma x_2(s_1 \Sigma x_1 + s_2 \Sigma x_2)^{-1} d]^{\frac{1}{2}}\]
\[= s_2[\zeta' G A'((A \Sigma_1 A')' s_1 + (A \Sigma_2 A')' s_2)^{-1} A \Sigma_2 A' (A \Sigma_1 A' s_1 + A \Sigma_2 A' s_2) A G' \zeta']^{\frac{1}{2}}\]
\[= s_2[\zeta' G A' (A \Sigma_1 A' s_1 + A \Sigma_2 A' s_2)^{-1} A \Sigma_2 A' (A \Sigma_1 A' s_1 + A \Sigma_2 A' s_2) A G' \zeta']^{\frac{1}{2}}\]
\[= s_2[\zeta' G A' (A \Sigma_1 A + I s_2) G A' (A \Sigma_1 A + I s_2) G A') A G' \zeta']^{\frac{1}{2}}\]
\[= s_2[\zeta' (A s_1 + I s_2)^{-1} (A s_1 + I s_2)^{-1} \zeta']^{\frac{1}{2}}\]
\[= s_2 \left[ \sum_{i=1}^{p} \frac{\zeta_i^2}{(s_1 \lambda_i + s_2)^2} \right]^{\frac{1}{2}} \tag{27}\]

Therefore, by the same arguments in (16) and (17), \(x_1\) is a monotonic increasing function of \(s_1\) and \(s_2\) is a monotonic decreasing function of \(s_1 (0 \leq s_1 \leq 1)\).

### 5.1 Ether \(x_1\) or \(x_2\) is given

I use same matrices in section (4).

Suppose \(x_2\) is given. If \(x_2 = x_2^*\), then \(x_2 = s_2(b'_x \Sigma x_2 b_x)^{\frac{1}{2}}\), where \(b_x = (s_1 \Sigma x_1 + s_2 \Sigma x_2)^{-1} d\). Therefore

\[x_2^* = (1 - s_1) \left[ \delta' A' \{s_1 A \Sigma_1 A' - (1 - s_1) A \Sigma_2 A'\}^{-1} A \Sigma_2 A' \right.\]
\[\left. \{s_1 A \Sigma_1 A' + (1 - s_1) A \Sigma_2 A'\}^{-1} A \delta \right]^{\frac{1}{2}} \tag{28}\]

Since \(x_2^*\) is known and \(x_2\) is a decreasing function of \(x_1\), I can easily approximate \(s_1\) and compute \(b_x = (s_1 \Sigma x_1 + s_2 \Sigma x_2)^{-1} d\).

Suppose \(p(1\mid 2) = 46.4\%\). In other words, \(\Phi(x_2) = 1 - p(1\mid 2)\) and \(x_2^* = 0.00913\). Since \(x_2\) is decreasing function of \(s_1\), I can approximate \(s_1 = 0.93\). Therefore, the optimum vector \(b_x\) is

\[b_x = \begin{pmatrix} -0.685 & -0.405 & -0.696 & 0.0881 \end{pmatrix}^T\]
\[x_1 = 0.6478 \quad \text{by (24)}\]

Therefore, the probability of misclassification \(p(2\mid 1)\) is

\[p(2\mid 1) = 1 - \Phi(x_1) = 1 - 0.85298 = 0.14702\]
5.2 Minimax procedure

Suppose \( x_1 = x_2 \). Since \( x_1 > 0 \) and \( x_2 > 0 \), \( x_1^2 = x_2^2 \).

\[
0 = x_1^2 - x_2^2 = s_1^2 b_x' \Sigma_x x_1 b_x - (1 - s_1)^2 b_x' \Sigma_x x_2 b_x
= s_1^2 b_x' A \Sigma_1 A' b_x - (1 - s_1)^2 b_x' A \Sigma_2 A' b_x
= b_x' A [s_1^2 \Sigma_1 - (1 - s_1)^2 \Sigma_2] A' b_x
\]

I can guess a value of \( s_1 \) and solve the quadratic equation for \( b_x \). I get \( s_1 \approx 0.534 \).

\[
x_1 = x_2 = 0.5369 \quad \text{by (24) and (25)}
\]

Therefore, the probability of misclassification \( p(1|2) \) and \( p(2|1) \) are

\[
p(1|2) = p(2|1) = 1 - \Phi(x_1) = 1 - 0.70433 = 0.29567
\]
5.3 Case of a priori probabilities and cost function

If I are given a priori probabilities, \( p_1 \) and \( p_2 \), and the cost functions, \( c(1|2) \) and \( c(2|1) \), the probability of a misclassification is

\[
p_1 c(2|1)[1 - \Phi(x_1)] + p_2 c(1|2)[1 - \Phi(x_2)]
\]

The optimum solution can be found if I solve the following equation.

\[
p_1 c(2|1)\Phi(x_1) \frac{\partial x_1}{\partial t_1} + p_2 c(1|2)\Phi(x_2) \frac{\partial x_2}{\partial t_1} = 0 \tag{29}
\]

However, there is no direct way to solve the differential equation.

5.3.1 \( \Sigma x_1 = k\Sigma x_2 \)

\[
\frac{p_1 c(2|1)}{(b_x' \Sigma x_1 b_x)^{\frac{1}{2}}} \Phi(x_1) = \frac{p_2 c(1|2)}{(b_x' \Sigma x_2 b_x)^{\frac{1}{2}}} \Phi(x_2) \tag{30}
\]

\[
(1 - s_1 - \sqrt{ks_1})(1 - s_1 + \sqrt{ks_1})b_x' \Sigma x_1 b_x = k \ln k + 2 k \ln \left( \frac{p_2 c(1|2)}{p_1 c(2|1)} \right) \tag{31}
\]

(1 - s_1 - \sqrt{ks_1})(1 - s_1 + \sqrt{ks_1})b_x' \Sigma x_1 b_x = k \ln k + 2 k \ln \left( \frac{p_2 c(1|2)}{p_1 c(2|1)} \right)

From (4.3.1) and (26), I know that LHS of the equation is monotonic increasing or decreasing in \( s_1 \) depends on the sign of \((1 - s_1 - \sqrt{ks_1})(1 - s_1 + \sqrt{ks_1})\).

There are ECM’s for the different \( k \)’s below.

| \( k \) | \( x_2 \) | \( x_1 \) | \( p(1|2) \) | \( p(2|1) \) | \( 0.5 \, p(1|2) + 0.5 \, p(2|1) \) |
|---|---|---|---|---|---|
| 0.3 | 0.03 | 1.10 | 0.4878 | 0.1359 | 0.3119 |
| 0.5 | 0.21 | 0.86 | 0.4166 | 0.1958 | 0.3062 |
| 1.0 | 0.58 | 0.58 | 0.2819 | 0.2819 | 0.2819 |
| 1.5 | 0.81 | 0.50 | 0.2104 | 0.3096 | 0.2600 |
| 2.0 | 0.96 | 0.48 | 0.1688 | 0.3169 | 0.2428 |
| 3.0 | 1.16 | 0.49 | 0.1240 | 0.3129 | 0.2185 |
| 4.0 | 1.28 | 0.51 | 0.0995 | 0.3042 | 0.2018 |
| 5.0 | 1.38 | 0.54 | 0.0841 | 0.2952 | 0.1896 |
5.3.2 \( \Sigma_{x1} = \Sigma_{x2} \)

\[
(1 - s_1 - \sqrt{ks_1})(1 - s_1 + \sqrt{ks_1})b'_x \Sigma_{x1} b_x = k \ln k + 2k \ln \left( \frac{p_2 c(1|2)}{p_1 c(2|1)} \right)
\]

\[
(1 - 2s_1)b'_x \Sigma_{x1} b_x = 0
\]

Since \( b'_x \Sigma_{x1} b_x \) is positive definite matrix, \( s_1 = \frac{1}{2} \). Therefore, replacing \( s_1 \) by \( \frac{1}{2} \), the equation (12) gives the optimal discriminant

\[
d = (s_1 \Sigma_{x1} + s_2 \Sigma_{x2})b_x
\]

\[
= (\frac{1}{2} \Sigma_{x1} + \frac{1}{2} \Sigma_{x2})b_x
\]

\[
= \frac{1}{2} (\Sigma_{x1} + \Sigma_{x1})
\]

\[
= \Sigma_{x1} b_x
\]

\[
b_x = \Sigma_{x1}^{-1} d
\]

The optimal vector \( b_x \) is same as the linear discriminant \( l_x = \Sigma_{x}^{-1} \mu_d \) (Arya et al., 2000).

6 Concluding Remarks

The educational purpose of this paper is providing accounting students with better understanding of the nature of accounting procedure. The preparation of financial statements is nothing but a linear process of accounting information aggregation. It is inevitable to lose information through the preparation process of financial statements (i.e. aggregation). The aggregation process provides benefits as well. One of the aggregation gains is related to the bounded rationality (Arya et al., 2000). More information may not be always optimal since the interpretation for overloaded information causes costs including time and money. Hence, many investors refer to audited financial statements for their decision making. Another potential benefit is related to the measurement errors. Measurement error in specific items may be canceled out through the aggregation process (Grunfeld and Griliches, 1960; Lim and Sunder, 1991; Datar and Gupta, 1994).

Accounting students can acquire better understanding of the mathematical implications of
accounting procedure from this exercise. Although this study provides general models for accounting discriminant analysis, next stage of development will likely extend this study in several directions. One case is that on average managers show equal mean matrix and unequal variance matrices in their transactions. To begin with, it will set up an agency model for the earnings management. I conjecture that investors (i.e. principal) can be better off from annual or quarterly reports (i.e. aggregated information) in the presence of volatility in the reported accounts of companies. There are two companies, managing earnings and non-managing earnings. Although two companies show same reported numbers in ending balance, investors can discriminated one from the other by checking the variance of accounts. In this regard, it would be worthwhile for future research to look into variability of accounting information over a longer interval.
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