Competition to Enter a Better School and Private Tutoring

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Abstract

In this paper, I build a choice-theoretic model which can explain private tutoring behavior which arises as an attempt to enter a better quality school. Students are assumed to differ in their ability and income, and schools in their quality. A better quality school confers students a better educational attainment. By taking private tutoring, a student can upgrade the quality level of the school of his/her choice. But there is a penalty for a mismatch between the school quality and student ability. As a result, a student with a given ability may not choose the highest quality school he/she can financially afford. An optimal amount of private tutoring is derived, and the equilibrium distribution of student ability in each school with a given quality is specified.

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1. Introduction

Students aspire to go to a better school, and study hard to win in the competition to get into better known, renowned schools\(^1\). So common a phenomenon is the aspiration to have a better quality good, whether the good in question is a house or an education, that there appears nothing special about the students’ craving for better schools. There exists, however, one significant difference between the competition to get into more prestigious schools and the competition to get higher quality ordinary goods. All you need to have to fulfill your long-cherished dream to live in a fancy apartment is more money. Better schools, however, cannot be bought with mere money in most cases. You need something more, or at the least, you need to convert the money you already have into something that is acceptable to the school you are eager to enter. That something else is, in most cases, a student’s academic achievement. Students are therefore, induced to study hard because studying hard is the only, or the most effective means to satisfy the school’s admission criterion.

This gives rise to a number of interesting questions. Why do schools refuse to accept money outright in their sale of educational goods, and instead asking students to transform it into some other form before they are allowed to be a rightful customer of the good? What would schools do to maintain their reputation, and thus continue to secure high quality students? How hard will students study to get into the desired

\(^1\) In every admission season, schools with a good reputation have more applicants than they can possibly accommodate. Consequently, those schools can be very selective in their admission decision, and, in the end successfully net out only those students with highest quality. This process repeats itself every year, as much to be called as a stable equilibrium in economists’ jargon.
school? Or, to put the same thing in slightly different words, how many resources would a student or his parents put into the required conversion process, etc.? In this chapter, I focus on the last two questions and attempt to build a simple choice-theoretic model which can shed some light on them.

In this paper, I call all the learning activity outside of the formal school, whether private or public, as private tutoring. It can, therefore, cover self-study at home, attending private educational institutions after school, hiring tutors to get help, etc., or anything that can be used to enhance the chance of being admitted to a better quality school. Generally speaking, students are supposed to study hard both in and out of the school, reviewing and previewing what they learn at school and practicing and applying it later on. Therefore, studying hard outside of school in itself is desirable thing and is recommended for every student.

However, sometimes there is a concern that students and their parents are put under too much burden by this ever increasing need for private tutoring. It is claimed that they are 'forced' to incur an exorbitant amount of expenses just to survive in the keen competition of the educational system. The need for extra study is especially acute where there is a severe competition to get in higher educational school, or where there is a price control and therefore an excess demand in the educational market. To obtain a good score in the entrance examination, students spend a tremendous amount of money and other resources, including students' own efforts. On a social level, this sometimes prompts an outcry for educational reform to correct the situation.

Korea, for example, has been known to have grappled with this so-called "excessive private tutoring" problem for long, with no great success during the last several decades. The list of anti-private-tutoring measures taken by the Korean government is impressively long and complicated. Reform after reform, new innovative measures were taken, from a simple admission policy reform to a drastic equalization of all
secondary schools in the country \(^2\). Japan, France and modern China also have a splendid history in this regard where private educational institutions are prospering. Once China also implemented an equalization measure, even though it quickly revoked it after a brief experimentation \(^3\). According to a recent report (Bray, 1999), private tutoring is prevalent in many countries, such as Sri Lanka, Malta, Egypt, Thailand, Mauritius, Hong Kong, Morocco, Tanzania, Zimbabwe and Taiwan.

Building a tractable model for private tutoring will therefore be of some help in expanding our understanding of the process, and possibly in devising better policy measures to cope with it. The model developed below is quite simple but rich enough to generate several interesting refutable hypotheses. The innovation introduced in the paper is to take notice of the fact that there exists a quality difference among schools and among students. On the one hand, the quality difference in schools create an incentive to go to better schools among students, because the quality differential is not, in most cases, correspondingly priced. On the other hand, the quality distribution of students creates the problem of securing homogeniety among students within a group who participate in the same educational process because an individualized care is, again in most cases, difficult and costly. In these circumstances, students do their best to maximize their utility, sometimes taking advantage of the under-priced quality differentials, and sometimes trying to make up for the loss they incur in the school by taking more private tutoring.

In this chapter, a special attention has been paid to an examination of the possible effects of the equalization measure taken in Korea in the 1970’s\(^4\). This is of particular interest especially because there is a close resemblance between what the Korean government has done 30 years ago and what the US government is currently trying to do under the name of "choice". In fact, the equalization measure is exactly the opposite to

\(^3\) For some illustrative stories for these countries, see Little and Wolf (1996), Dore (1997).
\(^4\) A little bit less formal a version has been presented in Chapter 3. Here we want to develop a more formal and rigorous analysis.
what the US government attempts to introduce these days: to equalize schools to reduce the 'excessive' competition among schools and students on the one hand vs., to allow more choices to promote and intensify the competition on the other hand. So, a close examination of the equalization measure may provide some hints on what will happen, in a negative way, if the choice program is successfully implemented in the US.

A growing number of papers are dealing with this quality and ability issue. Epple and Romano is one of them. Recently, in a quite influential paper, Epple and Romano (1988) develop a model in which school quality and student ability interact to generate many interesting reputable hypotheses on school choices, equilibrium distribution of school qualities, etc... In their model, however, there is nothing a student can do to increase his/her chance to enter a better school other than being born with high income and/or ability. Schools then select students based on their ability and income. In this chapter, I explicitly incorporate the efforts that must be exerted by students or their parents. Secondly, there is a penalty a student has to pay when there is a mismatch between school quality and student ability. Having a too high quality school can be bad not good.

The overall conclusion we derive from the model is that much of the "excessive private tutoring" is the natural outcome of the distorted pricing system in the educational market. The much touted "diploma disease" (Dore, 1997), or "obsessive pursuit of top-rank schools" are not the cause of the extortionate private tutoring but just symptoms of a misguided system. The disease, if it is a disease at all, is not in the people’s preferences but in the badly managed educational system. Another surprising result is that the equalization measure adopted with the sole purpose of reducing private tutoring is not likely to have had the desired effects. Rather, it may have aggravated the situation by inducing students to take more tutoring.

This chapter unfolds as follows. In section 2, a formal model is constructed and analyzed, and in section 3, the comparative static results
are presented. Section 4 investigates the effects of school equalization policy.

2. The Model

Students have a utility function, \( U(x,e) \), \( U_x > 0 \), \( U_e > 0 \), \( U_{ij} < 0 \), \( U_{ij} > 0 \) \((i \neq j)\) where \( e \) represents the amount of education they consume, and \( x \) all other goods. There are two different sources for educational attainment: one from formal school, \( e_1 \), and the other from private tutoring, \( e_2 \) and they are perfectly substitutable.

\[
e = e_1 + e_2
\]

(1)

The school education is assumed to be provided for free by the government. This is a subsidy in kind. Since \( e_2 \) refers to the learning activity pursued outside of the schools, I do not have the problem of mutual exclusivity that appears in some of the literature on the relationship between public and private education (Pelzman, 1973 for example)\(^5\). In the model, all school education, whether public or private, is represented by \( e_1 \) and is assumed to be provided for zero price.

The amount of education that an individual student can get from school is given by the following function:

\[
e_1 = \bar{e}_1 + \alpha \hat{e}_1 (q) - \beta (a - q)^2, \quad \alpha, \beta > 0, \quad \hat{e}_1 (0) = 0, \quad \hat{e}_1 ' > 0, \quad \hat{e}_1 '' < 0
\]

(2)

That is, the school education is composed of two parts: one is \( \bar{e}_1 \) which every student gets invariably from school, and the other, which is related with the school quality, \( q \), and student's individual ability, \( a \). A student who chooses a high-quality school gets a larger amount of education, but has to pay a penalty if his/her ability is far from the quality level of

\(^5\) If \( e_1 \) and \( e_2 \) represent two different 'school' educations each, a student who is enrolled in one school can not be enrolled at the other school simultaneously.
the school he/she attends. There exists a lower limit of $q$, $a \leq q$, which is assumed to be zero for convenience $^6$. The minimum quality education is guaranteed for every student.

Getting into a high quality school is costly. The cost is paid in the form of $e_2$. Studying hard outside the school will improve his chance of getting into a high quality school.

$$q = \psi(e_2), \quad \psi > 0, \quad a = \phi(0) \quad ^7.$$ (3)

The price of $e_2$ is $p$, $(p > 0)$. In this sense, the school quality is not free at all. In the other sense which will become clearer shortly, however, it is sometimes free.

As far as major building-blocks are concerned, these complete the model. Before formally presenting the maximization problem with which a student is faced, however, there remains one thing to note.

Note that private tutoring in the above setting has a dual function. On the one hand, it is consumed in itself. Purchasing more $e_2$ therefore implies just having more education. On the other hand, it can be used to increase the probability of getting into a better school, and thereby to secure the chance of increasing his/her education still further$^8$. Note also that use of $e_2$ in either one of the purposes does not exclude the usage of the same thing for the other purpose. In this sense, private tutoring in the model has the typical property of public goods: the amount of $e_2$ used for one purpose does not reduce the amount of $e_2$ left for the other

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6) In reality, there will also be a upper limit. But the analysis is analogous to the case of lower limit.
7) To be more realistic, we may assume $q = \Psi(e_2)$, $\Psi_t$, $\Psi_{\psi} > 0$, $\Psi_{\phi} > 0$. This, however, would change none of the major results.
8) A student may have a higher chance of advancing into an honour class or transferring into a more prestigious school with a good reputation. In some countries where secondary school is divided into two levels, junior middle and high school, he/she may have a better chance of getting into so-called top-notch high schools. Or the current setting can be understood to refer to the whole life-time education in which there are several chances to switch the tracks of educational pursuit.
Furthermore, it is freely disposable in each usage 10). Even if a very prestigious top-quality school offers admission to a student, he/she can reject it if he/she considers the school unfit compared to his/her own ability. A student has to buy if he/she wants more of school quality than he/she currently has, but he/she can dispose of with no additional cost if he/she has enough of it already.

This dual nature of private tutoring will require us to reformulate the variables and model in a slightly different way in the following. But before doing that, I will first examine the problem of choosing the optimum level of school quality in the current setting, and then later discuss the problem of choosing the optimum amount of private tutoring in a slightly altered setting.

Problem of School Choice

Let’s first take note of a special characteristic of the school education function given above in (2).

**Proposition 1:** For a given level of school quality, there exist a lower bound and an upper bound of student ability in such a way that students whose ability level lies outside of these bounds will not attend the school at all even if the school education is freely provided.

9) Dual functions played by a certain type of education appear in other settings too: for example, in the signaling model. There, education enhances individual productivity, and, at the same time, plays a role as a signal to potential employers who lack an exact information about the applicant’s intrinsic ability.

10) The fact that a good can be used for many purposes simultaneously does not itself guarantee free disposability. An immediate example is our $e_1$ function within which $e_2$ has again two roles to play: one role of increasing the amount of school education by improving the school quality, $a e_1(\phi(e_2))$, and the other increasing or decreasing the penalty a student has to pay when the ability and quality diverges, $\|b - \phi(e_2)\|$. Nonetheless, you cannot choose a different amount of $e_2$ for each term because they are not freely disposable in this setting. The penalty, if any, can be disposed of only by changing the $e_2$ exactly by the same amount at the same time.
**Proof:** The school education function has a quadratic term in $a$ and $q$, implying that $e_1$ can be negative if the gap between $a$ and $q$ is large. Since a negative education cannot be imposed upon students, this implies that some students will opt or drop out of school. More concretely, for $e_1$ to be non-negative, we need,

$$q - \sqrt{\frac{e_1 + a \hat{e}_1(q)}{\beta}} \leq a \leq q + \sqrt{\frac{e_1 + a \hat{e}_1(q)}{\beta}}.$$ (4)

Let these bounds be denoted as $a(q)$, and $\overline{a}(q)$, each. Students outside of this range quit attending the school because the penalty they have to pay for the mismatch between the quality and ability is too high. For them, school education is not a good but a bad.

In short, a school with a given quality level can serve only a limited range of students. Probably it may explain why we have a hierarchical school system in every society, in which schools are graduated into some sort of ascending order, such as kinder-garten, elementary, secondary schools, and colleges. In addition to that, for those students who are extraordinarily talented or slow, we provide separate educational institutions specifically targeted for them only. In that regards, the parametric values in the above expression must be interpreted to pertain only to a particular hierarchical school level.

For a later reference, note that the inequality in (4) can be restated in terms of $q$, implying that there is a lower and an upper limit of school quality outside of which a student with a given ability level will never choose.

Earlier it was assumed that school quality can be increased only by taking private tutoring. In order to prepare ourselves for the ensuing

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11) This does not necessarily mean that students will actually stay out of school outright. Some students may continue to show up at school under the familial or societal pressure, but doing nothing or other things in the class.
analysis, however, let’s first consider a hypothetical case where students can directly choose school quality and look into the relationship between the school quality and student ability. Here, it is temporarily assumed that the price of quality is given by \( q \) (\( q \geq 0 \)), which implies that a student has to incur a proportionately higher cost as he/she chooses a better quality school\(^{12}\). At the same time, it is also assumed that students are willing and able to pay \( q \) \(^{13}\).

Under these assumptions, we can immediately establish the following propositions.

**Proposition 2:** there exists a unique school quality level that a student with the ability index \( a \) will choose for the price \( q \),

\[
q^*(a,q) = a + \frac{a \hat{e}_1(q)}{2q} - \frac{\hat{\delta}}{2q},
\]

(5)

with \( q^*_a > 0 \) and \( q^*_q < 0 \).

**Proof:** The proof follows by itself when we apply the standard maximization method to the problem of

\[
\text{Max}_q \quad e_1(q) - \delta q.
\]

Let \( \frac{de_1}{dq} = a \hat{e}_1 + 2\beta(a-q) = \xi(q) \). Then the FOC is \( \xi = \delta \). Since \( \xi \) is monotonically decreasing (\( \xi' < 0 \) because \( \hat{e}_1'' \leq 0 \)), and \( \delta \) is constant, a unique solution, \( q^*(a,\delta) \) exists. Differentiating the FOC with respect to \( a \) and \( \delta \), respectively, we have \( q^*_a = -\xi'/\xi' > 0 \), \( q^*_q = 1/\xi' < 0 \).

Above, the first term of \( \xi \) captures the effect of school quality improvement due to an increased \( e_2 \), and is thus positive. The second term captures the penalty effect, and is positive for those students whose

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\(^{12}\) Given \( q = \Psi(e_0) \), the price of \( q \) will be equal to \( \delta = \Psi'/\rho \).

\(^{13}\) Obviously, willingness and ability to pay is determined by the interaction of the marginal rate of substitution, income and price, which will be incorporated later.
ability is greater than the school quality, and negative for those students whose ability is lower than the school quality. Combining these two effects, we can see that $\xi(q)$ function declines as $q$ increases, cutting the horizontal axis at $a + a \frac{\partial \xi}{\partial p} / 2 \beta$ and then sinking into the negative range\textsuperscript{14}. The expression $q^*_s > 0$ says that, with other things being equal, a better-abled student will choose a higher quality school, and the expression $q^*_s < 0$ implies that students will buy less quality, the higher the price is.

Let the quality level to be chosen under the zero price be denoted as

$$q^\sim(a) = \{q | \xi(q) = 0\} = q^*(a,0). \quad (6)$$

Since this is the level of school quality that a student will choose under zero price, it can be called a saturation quality level. Later, it will be shown that some students will pay virtually a zero price for school quality and become saturated, and others pay a positive price and be constrained by it. In the meantime, it is easy to check from (4) that, under zero price, a student will pick a school whose quality level is slightly higher than his/her ability level\textsuperscript{15}. This optimum level of school quality will be higher, the larger $a$ is and the smaller $\beta$ is. That is, the more effective school quality is in increasing the amount of education a student can get, the more he/she will purchase. Similarly, the smaller the penalty that a student has to pay for the mismatch between quality and ability is, the higher quality he/she will choose.

Since $q^*$ is monotonic in $a$ ($q^*_{a>0}$), we can invert this function to have

\textsuperscript{14} Strictly speaking, we need a condition, $a + a \frac{\partial \xi}{\partial p} (0)/2 \beta > 0$ to validate this statement, which we assume in the paper.

\textsuperscript{15} Through education, individual ability will improve eventually. Here, it is assumed to be given at the time when he/she makes a choice on the level of school quality.
\[ a^*(q) = q - \frac{a \hat{e}_1'}{2\rho} + \frac{\hat{e}_2}{2\rho}. \] (7)

This can be interpreted as the ability level of those students who will choose a particular level of \( q \) as their optimum. In the following, I will use \( a^*(q) \) to denote the ability level when \( \delta > 0 \), and \( a^-(q) \) when \( \delta = 0 \). A similar notational rule will also be applied to \( q \) and \( e_2 \) when there is a need to make that distinction.

With a positive price for school quality, the optimum quality level chosen will always fall short of the corresponding saturation level, i.e., \( q^* - q^* < 0 \), with the gap between the two ever widening as \( \delta \) gets higher. In an extreme case in which the price of private tutoring is extremely high, the desired \( q^* \) may be too low for most of the students, way below the lower limit of the quality that a given ability student would ever choose. This is obviously a least interesting case, which I rule out in the following\(^{16}\).

Now I introduce the school of minimum quality freely provided to every student into the picture.

**Proposition 3:** Students whose desired school quality, \( q^*(a) \), is lower than the freely provided minimum, \( a_0 \), will attend the minimum quality school with no additional investment to enhance the quality. Students whose desired quality is higher than the minimum will make a positive investment to improve the quality and, therefore, incur a positive amount of cost.

Or, equivalently, students whose ability index lies within the range of

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\(^{16}\) Since private tutoring in our model contains all the learning activities that take place out of school, including self-study at home, the price may not possibly go that high because of the close substitutability among the variety of activities that compete with each other. Furthermore, some schools may offer scholarships to highly talented students, thus lowering in effect their \( \delta \). The present model, however, does not allow this kind of price discrimination because schools cannot control the price of private tutoring directly.
\( a(q) \leq a \leq a^*(q) \) will attend the school of minimum quality with no investment. Students with \( a > a^*(q) \) will make a positive investment to improve the quality.

Proof: Note that the optimum quality \( q^*(a) \) that a student would voluntarily choose can be lower than \( q \), the minimum quality provided freely by the government. There are two distinct cases under this category. In one case, the minimum quality freely provided is genuinely too high for some of the students whose ability is low, i.e., for those students whose ability lies within the range of \( a(q) \leq a \leq a^*(q) \). They would have chosen a still lower quality school if such a school were available. Unfortunately, no such schools are available. Therefore, they have no choice but to attend a school whose quality level is undesirably high from the individual standpoint. The excess quality in this case is not freely disposable because of the absence of alternatives. Thus they attend, with some grudge, the school with \( q^17 \). The second group comprises of those students whose ability index falls within the range of \( a^*(q) < a \leq a^*(q) \). For them a school quality higher than the minimum is preferable, but they cannot afford to buy any additional quality because the price is too high for them. So, they also stay at the minimum quality school. Lastly, for the students with \( a > a^*(q) \), the minimum quality is not binding, compared to both their ability and price, so they will choose \( q^*(a) \) with some positive investment\(^18\).

Let’s assume, again temporarily, that not only the lower bound, but also an upper bound of school quality exists below which a student can freely choose any quality but must incur the capacity expansion cost if he/she wants to have a quality beyond that level. In this case, we can establish the following proposition which states that within a certain range, school quality becomes freely disposable.

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17) They can, however, still get some positive amount of education by staying within the school, rather than dropping out completely.

18) For a formal proof, we need to set up a maximization problem similar way to the one used for the proof of Proposition 3 below. We have omitted it here to avoid duplication.
Proposition 4: (Free disposability) In the case where there is a quality capacity limit, $\bar{q}$, which can be expanded by paying a positive price, $\delta$, a student will either choose $q^*(a)$ if he/she is not bound by the limit, or will optimally expand his/her $\bar{q}$ with a positive payment if he/she is bound by the capacity constraint.

Proof: Again this is the standard result we would get when solving the following maximization problem,

$$\max_{q, \bar{q}} \quad e_1(q) - \delta \bar{q} \quad \text{subject to} \quad q \leq \bar{q}$$

(8)

by setting the following Lagrangian equation,

$$L = e_1(q) - \delta \bar{q} + \lambda (\bar{q} - q).$$

(9)

The FOC is:

1) $L_q = e_1' - \lambda = 0$
2) $L_{\bar{q}} = \lambda - \delta = 0$
3) $L_{\lambda} = \bar{q} - q \geq 0, \quad \lambda \geq 0$ and $\lambda (\bar{q} - q) = 0$

From 3), if $q \leq \bar{q}$, then $\lambda = 0$. Plugging into 1), we have $e_1' = \xi = 0$, yielding $q = q^*$. If, on the other hand, $\lambda > 0$, then $q = \bar{q}$. In this case, from 1) and 2), $e_1' = \xi = \lambda = \delta$. Therefore, we have $q = \bar{q} = q^*(a, \delta)$.

The following graph (Figure 5-1), which shows the relationship between school quality, $q$, and individual ability, $a$, sums up the major conclusions of the above discussion in an $(a, q)$ plane.

Note that the slope of the indifference curve associated with a given amount of $e_1$ is given by the following equation,
\[
\frac{dq}{da} = \frac{2\beta(a-q)}{\xi} = \frac{1}{a \hat{e}_1(q) + 1}.
\]

(10)

Therefore, the slope is zero along the line, \( q=a \), implying that for a given \( q \), \( e_1 \) is maximized when a student’s ability is exactly matched with the school quality level, and any deviation from it entails a penalty.

On the other hand, along the line \( q^* = a + \frac{a}{2\beta} \hat{e}_1(q^*) \) \(^{19}\), the slope of the indifference curve is infinite, implying that for a student with a given ability level, \( e_1 \) is maximized when he chooses a school with its quality level slightly higher than his individual ability.

\(^{19}\) In a general setting, it is not a line but a curve with a slope,

\[
\frac{dq}{da} = \frac{1}{1 - \frac{a}{2\beta} \hat{e}_1''(q)}. 
\]

As \( q \) gets higher, the slope approaches 1 starting from some value less than 1. Of course, it can be 1 everywhere if \( \hat{e}_1''(q)=0 \).
When the minimum quality is given at $q$, a student with ability $a_i$ (= $q=0$) will get the highest $e_1$ among all students who stay at that school if he/she, too, decides to stay there. He/She will not, however, remain there, but move to a school with $q^\sim(a_i)$ to increase his school education if school quality is free ($\delta=0$). If the school quality is not free, but has to be bought for a positive price ($\delta>0$), he/she can not go all the way to $q^\sim(a_i)$, but has to stop at $q^*(a_i)$. In either case, if a school with a quality higher than $q^\sim(a_i)$ offers him/her an admission for free, he/she will simply reject it. Consequently, the area comprised of the points above
the line \( q^* = a + \frac{\alpha}{2\beta} \hat{e}_1'(q) \) and another line \( q = a \) will become "the area of free disposability."

Note also that for a student with ability \( a_e \) (=\( a^e(a) \)), the minimum quality provided is identical to the quality level he/she would have voluntarily chosen. This implies that any students whose ability is lower than \( a_e \) (but higher than \( a_1 \), about which I will explain shortly) will attend the minimum quality school somewhat reluctantly. As explained earlier, they would have chosen a still lower quality school than \( a \) if such a school exists. With no such school in existence, they stay at the lowest available school out of necessity, not out of free choice. The excessive quality in this case is represented by the vertical distance between the two lines, \( q^* \) and \( a \).

For those students who fall within the range of \( a_e \) and \( a_1 \) (where \( q^* \) line meets with \( a \) line), any quality higher than \( a \) but lower than \( q^* \) is preferred, but unaffordable. Thus, these students will also hang around at the minimum quality school with no purchase of an additional quality.

Every student whose ability is greater than \( a_1 \) purchases a positive amount of private tutoring and chooses his/her own quality school. The amount of quality purchased is represented by the vertical distance between the \( q^* \) line and the horizontal line \( a \). Since \( q^* \) is strictly increasing in \( a \), no students with different abilities share the same quality school here. In this sense, there is a perfect sorting of student abilities across the schools \(^\text{20}\). However, this is because we have implicitly assumed in this section that students are not bound by their income and preferences. All students, therefore, behave as if they have

\(^{20}\) If there is a perfect sorting of student ability across schools, the penalty term in our school education function will lose much of its significance since each school in this case can adjust its teaching level, say, the curriculum or the pace of progress, to the given ability level of the perfectly homogeneous student body, unless the adjustment itself is very costly. We will not have a perfect sorting, however, for the following two reasons. First, in the real world, we do not have a continuum of school qualities. Thus, the sorting will necessarily be imperfect. The second reason is explained in the text.
exactly the same income and the same marginal rate of substitution between education and other goods. They differ only in their abilities from each other. Consequently, this difference in ability has been monolithically mapped into a different demand for school quality. This conclusion will be altered if we introduce a utility function and income in our formal analysis, as we do in the ensuing section.

Students whose ability is lower than \( a_1 \) will choose not to attend any school at all. For them, attending school generates sheer pain, not benefits. For that matter, students with ability higher than \( a_5 \) would have also chosen not to attend school if the school with \( q \) were the only school available. Fortunately, however, they have higher quality schools to choose and don’t need to worry, unless they are really hard pressed by the price constraint\(^{21}\). The thick solid indifference curve passes through the origin and represents the amount of school education which is just equal to \( e_1 \).

\(^{21}\) As \( \delta \) gets higher, the \( q^* \) line shifts down continuously. This will make the \( a_3 \) point (the intersection between \( q^* \) and \( q \)) move to the right. If \( \delta \) is sufficiently high, \( a_3 \) will coincide with \( a_5 \) forcing every student below \( a_6 \) to stay at the lowest quality school. Outside of \( a_5 \), some students may or may not purchase some \( q \) (and attend a school higher than the minimum) depending on whether \( q^* \) line cuts the indifference curve from below or from above. This in turn depends on the curvature condition of the \( e_1(q) \) function. All these things, however, are logically possible, but quite unlikely for the reasons explained earlier. See footnote (136).
Problem of Choosing Optimum Private Tutoring

So far, we have analyzed how a different level of school quality is chosen by students, assuming, first, that the quality level itself is a control variable students can directly maneuver, and second, that all the students have the same income and preferences. But neither of them is true in our model, and we have to correct the situation in this section. This can be done by explicitly introducing our second variable of education, \( e_2 \), and the utility function both of which have so far been put aside temporarily.

A formal introduction of private tutoring variable at this juncture, however, requires us to simultaneously consider the dual effect of private education aforementioned, i.e., a freely disposable public good within the capacity. In order to do this, I reformulate the school choice problem a student is faced with in the following way. First, imagine a situation where a student is endowed with a certain amount of \( e_2 \), denoted as \( \bar{e}_2 \), which will play the role of capacity limit in subsequent decisions. The question of how this endowment is determined will be discussed later. The student then uses this \( \bar{e}_2 \) for two different purposes: one for changing the school quality and the other for increasing his/her own consumption. Let the amount of \( e_2 \) used for the former purpose be denoted as \( e_{2i} \), and the amount for the latter purpose as \( e_{22} \). As long as \( e_{2i} \leq \bar{e}_2 \) (i=1,2), he/she can freely assign any amount of \( e_2 \) for either purpose. Now go back to the initial question of how this endowment is pinned down. Simply put, it has to be purchased for a price. When he/she makes a decision on \( \bar{e}_2 \), however, he/she correctly foresees all these embedded features in advance, and sets it at the optimal level\(^{22}\).

We are now ready to set up the full-blown maximization problem a representative student has.

\(^{22}\) This is similar to the peak-load pricing model. See Steiner (1964).
\[ \begin{align*}
\max_{e_2, e_2^\mu, e_2^\nu} & \quad U(x, e) \quad \text{subject to} \quad e = e_1 + e_2 \\
& \quad e_1 = \overline{e}_1 + a \overline{e}_1 (q) - \beta(a - q)^2 \\
& \quad q = \Psi(e_2^\mu) \\
& \quad y = x + p \overline{e}_2 \\
& \quad e_1 \geq 0, \quad \overline{e}_2 \geq 0 \\
& \quad 0 \leq e_2 \leq \overline{e}_2, \quad 0 \leq e_2^\mu \leq \overline{e}_2. 
\end{align*} \] (11)

Above, the fourth equation is the ordinary budget constraint where the price of \( x \), the numeraire good, is normalized to be 1. The last two inequalities say that the student cannot sell his/her \( e_1 \) and convert it into more of \( x \). Nonetheless, a student can control the actual amount of \( e_1 \) he/she consumes by choice of \( e_2 \), unlike the conventional subsidy-in-kind model where \( e_1 \), once given, is fixed.

This set of equations gives rise to the following Lagrangian function,

\[ Z = U(x, \overline{e}_1 + a \overline{e}_1 [\phi(e_2^\mu)] - \beta(a - \phi(e_2^\mu))^2 + e_2) \]
\[ + \lambda(y - x - p \overline{e}_2) + \mu_1(\overline{e}_2 - e_2^\mu) + \mu_2(\overline{e}_2 - e_2). \] (12)

Here \( \mu_i \) has the meaning of the marginal utility of capacity expansion for either purpose. Note that the student has three control variables to optimize, \( \overline{e}_2, e_2^\mu \) and \( e_2^\nu \), all of which are the derivatives from \( e_2 \).

The Kuhn-Tucker first order conditions are then,

\[ Z_x = U_x - \lambda = 0 \] (13)
\[ Z_{e_2} = U_e - \mu_1 \leq 0, \quad e_2^\mu \geq 0, \quad e_2^\mu Z_{e_2} = 0 \] (14)
\[ Z_{e_2^\nu} = U_e - \mu_2 \leq 0, \quad e_2^\nu \geq 0, \quad e_2^\nu Z_{e_2^\nu} = 0 \] (15)
\[ Z_{\overline{e}_2} = -\lambda p + \mu_1 + \mu_2 \leq 0, \quad \overline{e}_2 \geq 0, \quad \overline{e}_2 Z_{\overline{e}_2} = 0 \] (16)
\[ Z_d = y - x - p \overline{e}_2 = 0 \] (17)
\[ Z_{\mu_1} = \vec{e}_2 - e_{21} \geq 0, \quad \mu_1 \geq 0, \quad \mu_1 Z_{\mu_1} = 0 \]  
\[ Z_{\mu_2} = \vec{e}_2 - e_{22} \geq 0, \quad \mu_2 \geq 0, \quad \mu_2 Z_{\mu_2} = 0. \]  

where \( \eta = \frac{d e_{21}}{d e_{21}} = [\alpha \vec{e}_1 \cdot + 2 \beta (a - q)] \psi = \xi \psi \), which yields the following series of lemmas.

**Lemma 1:** Private tutoring has the property of public goods.

**Proof:** From (13) and (16), we can immediately establish

\[
\text{if } \frac{\mu_1 + \mu_2}{P} > u_x, \text{ then } \vec{e}_2 > 0.
\]

otherwise, students will purchase a zero amount of \( \vec{e}_2 \).

That is, marginal utilities in two different uses are summed together to be compared with its marginal cost.

Lemma 1 holds for any public goods, whether freely disposable or not. Thanks to the free disposability assumption, however, we can establish a much stronger lemma.

**Lemma 2:** Private tutoring is worthwhile to take if it is worthwhile for only either one of the purposes.

**Proof:** \( \frac{\mu_1 + \mu_2}{p} \geq \frac{\mu_i}{p} \) (i=1 or 2) because \( \mu_i \geq 0 \).

Conceptually, we may have four different possible combinations of \( \mu_1 \) and \( \mu_2 \) depending on which one takes 0 or a positive value. Out of those four, we can rule out the two cases in which \( \mu_2 = 0 \).

**Lemma 3:** \( \mu_2 > 0 \) always, i.e., the consumptive demand for \( e_2, e_{22} \) is always bound by the capacity limit, \( \vec{e}_2 \).
Proof: From equation (15) and our assumption $U_c > 0$, $\mu_2 \geq U_c > 0$. Then, from (19), $e_{22} = \tilde{e}_2$ because $\mu_2 > 0$.

Unlike $\mu_2$, $\mu_1$ can be either 0 or positive, as shown below.

Lemma 4: Quality upgrading demand for $e_2$, $e_{21}$ is either bound by $\tilde{e}_2$ or unbound. When it is bound, $e_{21}^* = \tilde{e}_2$. When it is unbound, the optimum amount of purchase will be either $e_{21} = 0$ or zero, where $e_{21} = \{e_{21} \mid n(e_{21}) = 0\}$.

Proof: From equation (18), if $\mu_1 > 0$, then $e_{21} = \tilde{e}_2$. From equation (14), if $U_\eta - \mu_1 < 0$, then $e_{21} = 0$. If $e_{21} > 0$, then $U_\eta - \mu_1 = 0$. If $\mu_1 = 0$, $e_{21} < \tilde{e}_2$. This implies $U_\eta = 0$, which requires $\eta = 0$. Therefore $e_{21} = e_{21}^*$.

So far we have checked whether $e_{21}$ and $e_{22}$ are bound by $\tilde{e}_2$ and found that $e_{22}$ is always bound, and $e_{21}$ may or may not be bound. $\tilde{e}_2$ can, however, still be equal or greater than 0. This implies that we can have the following two cases: one in which both $\mu_1$ and $\mu_2$ are binding, and the other in which only $\mu_2$ is binding.

Case 1) Both $\mu_1$ and $\mu_2$ are binding.

In this case, we have interior solutions for both $e_{21}$ and $e_{22}$ such that $e_{21} > 0$ and $e_{22} > 0$. Then equations (14) and (15) will hold with equality.

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23) Caveat! $\mu_2 > 0$ doesn't mean that $e_{22}$ is worthwhile to take. What the lemma 2 says is that if $\mu_2/p$ is greater than $U_\xi$, then $(\mu_1 + \mu_2)/p > U_\xi$. A mere positive value of $\mu_2$ does not, however, guarantee that it will be greater than $U_\xi$. Theoretically speaking, conditions for having interior solution and conditions for free disposability are two different things. $\mu_2 > 0$ implies no slacks or complete exhaustion of existing capacity and the statement that something is worthwhile to take implies interior solutions.

24) $\eta(e_{21}) = 0$ implies $\zeta(e_{21}) = 0$ since $\eta = \zeta \Psi$ and $\Psi > 0$ always.

25) From our implicit assumption, we have $\lambda > 0$, $x > 0$ already. Therefore we have equations (13) and (17) with equality.
Lemma 5:

If \( \frac{U_e}{U_x} < \frac{p}{\eta+1} \), then \( \bar{e}_2^* = 0 \). \hspace{1cm} (20)

Otherwise \( \bar{e}_2^* > 0 \) and at equilibrium, \( \frac{U_e}{U_x} = \frac{p}{\eta+1} \). \hspace{1cm} (21)

Proof: Similar to the proof to lemma 4, thus omitted.

Equation (20) says that a student whose marginal rate of substitution (MRS) between x and e at \( \bar{e}_2=0 \) is smaller than \( p/(\eta+1) \) will opt to have zero amount of \( \bar{e}_2 \), and, therefore, all \( e_2 \)'s are zero.

Case 2) Only \( \mu_2 \) is binding.

Lemma 6: When only \( \mu_2 \) is binding, \( \bar{e}_2 > 0 \) always. In this case, \( e_2 = 0 \) or \( e_2 = \bar{e}_2 \). The optimum \( e_2 \) is achieved at \( U_e = U_3 \).

Proof: On the one hand, the fact that \( \mu_1 = 0 \) implies \( e_2 < \bar{e}_2 \) in (18) and \( U_e \eta \leq 0 \) in (14). There are two cases: either \( \eta < 0 \) or \( \eta = 0 \). If \( \eta < 0 \), then \( e_2 = 0 \). If \( e_2 > 0 \), then it must be \( \eta = 0 \) to have \( U_e \eta = 0 \). It implies \( e_2 = e_2 = \bar{e}_2 > 0 \). In either case, \( \bar{e}_2 > 0 \). On the other hand, \( \mu_2 > 0 \) implies \( e_2 = \bar{e}_2 \), which is positive, and thereby establishes \( U_e = \mu_2 \) in (15) and \( \mu_2 = U_3 \) in (16). Putting together, \( U_e = U_3 \).

In this range, students have already a high enough school quality and any addition to it will generate a pain. Thus, they stay at their best school with the saturation level of quality, without being forced to pay the penalty associated with the un-wanted higher quality school. In short, they will freely dispose of whatever redundant quality. They will, however, purchase whatever amount of \( e_2 \) without worrying about the bads it would generate because of this free disposability.

Putting all these lemmas together gives rise to the following
Proposition 5: The maximization problem posed in (11) has the following three solutions: one corner solution and two interior solutions.

1) Corner solution:

\[ \bar{e}_2 = e_{21} = e_{22} = 0 \quad \text{if} \quad \frac{U_e}{U_x} < \frac{p}{\eta + 1} \quad \text{at} \quad \bar{e}_2 = 0 \quad \text{(with the constraint } \eta \geq 0) \]

2) Interior solution I:

\[ \bar{e}_2 = e_{21} = e_{22} = e_2^* > 0 \quad \text{if} \quad \frac{p}{\eta + 1} \leq \frac{U_e}{U_x} < p \quad \text{at} \quad \bar{e}_2 = 0 \]

where \( e_2^* = \{ e_2 \mid \frac{U_e(e_2)}{U_x(e_2)} [z(e_2) + 1] = p \} \)

3) Interior solution II:

\[ \bar{e}_2 = e_{21} = e_{22} = e_2^* \quad \text{and} \quad e_{21} = e_2^* \quad \text{if} \quad \frac{U_e}{U_x} \geq p \quad \text{at} \quad \bar{e}_2 = e_2^* \]

where \( e_2^* = \{ e_2 \mid \frac{U_e(e_2)}{U_x(e_2)} = p \} \quad \text{and} \quad e_2^* = \{ e_2 \mid z(e_2) = 0 \} \).

The following graph gives us an intuitive illustration on the two interior solutions (Figure 2). As explained, a given \( e_2 \) is used for two different purposes at the same time. When it is used for direct consumption, the MRS is given by \( U_e/U_x \). When it is used for quality improvement, the corresponding MRS is given by \( \eta U_e/U_x \). The total demand for private education is then the vertical sum of these two curves, with the proviso that the negative part of \( \eta U_e/U_x \) curve must be discarded and replaced with 0 value. The thick solid curve in each panel represents this sum. At equilibrium, marginal value of private education must be equated to its price, and equilibrium is found at \( e_2^* \), in each panel.
Note that in panel (a) which depicts the Interior Solution I, $e_2^*$ is located to the left of $e_2^-$, the satiation point, which is the solution to $\eta\left(\frac{U_{x}}{U_{x}}\right)=0$ 26). Here, both $\frac{U_{x}}{U_{x}}$ and $\eta \frac{U_{x}}{U_{x}}$ are binding, and, therefore, we have $e_2^*=e_2^-=e_2^*=\left\{ e_2 \mid \frac{U_{x}}{U_{x}}=\eta/(n+1) \right\}$. In panel (b) which represents the Interior Solution II, however, the equilibrium solution $e_2^*$ is located to the right of $e_2^-$ 27). This implies that only $U_{x}/U_{x}$ is binding, and, therefore, we have two different solution values for $e_2^*$ one for quality improvement, $e_2=e_2^-$ and the other for direct consumption, $e_2=e_2^*=\left\{ e_2 \mid \frac{U_{x}}{U_{x}}=\eta \right\}$.

26) Or, to put the same thing in different words, $e_2^*$ is located in the range where $\eta \frac{U_{x}}{U_{x}}$ is positive.
27) Or in the range where $\eta \frac{U_{x}}{U_{x}}$ is negative, and thus the redundant quality is freely disposed of.
Note that in panel (a), the availability of a better quality school has caused the student to have more private tutoring. With no school choice, he/she would have chosen an $e_2'$ amount of private education. He/She is, however, now choosing $e_2^*$ because he/she can upgrade the quality of the school with private tutoring. In contrast, the availability of quality school in panel (b) does not affect the student's demand for private education because his/her consumptive demand for education already exceeds the demand for school quality. In short, the equilibrium value of $e_2$ is invariant to the location of the $\eta U_e/U_x$ curve.

Analogously we can depict the corner solution in a similar way. (See Figure 3 below). We already know that there are two different cases in which students decide not to purchase private tutoring: they do not buy either because the price is too high, or because the minimum they have is already high enough. Panel (a) represent the first case, and panel (b) the second. Note that a zero purchase of $e_2$ corresponds to the minimum school quality $q$. In panel (a), compared to the price, the sum of $U_e/U_x$ and $\eta U_e/U_x$ is so low that the student chooses not to purchase any private tutoring, even though he/she still wants to have more ($e^\sim$ is positive).

In panel (b), $e_2^\sim$ is negative, implying that his/her preferred level of school quality is lower than the minimum provided. Therefore, some of the quality is redundant for him/her, but unlike all other cases discussed so far, he/she cannot throw it away, because it is not freely disposable any longer. Graphically, it means that the $U_e/U_x$ and $\eta U_e/U_x$ curves now have to be vertically summed as they are without the negative part of $\eta U_e/U_x$ being slashed off. In the positive range, however, the free disposability of $\eta U_e/U_x$ gets back to work and the negative part must be cut off. This implies that at the vertical intercept, there arises a discontinuity in $U_e/U_x+\eta U_e/U_x$ curve because it is switched from the non-free disposal range to free disposal range when it crosses the vertical axis.
The following proposition concerns the effective price that a student is paying in the first interior solution.

**Proposition 6:** The "effective shadow price" for \( e_2 \), \( p/(\eta+1) \), is lower than its nominal price, \( p \), for those students whose \( \eta \) is greater than 0 at equilibrium (or equivalently \( \eta U_e/U_x \) is binding at equilibrium).

**Proof:** This is because a student who has purchased a positive amount of \( e_2 \) can enjoy a still larger amount of education from the better school he/she is made able to attend without incurring additional cost. The purchase of one unit of \( e_2 \) generates its own utility and, in addition to that, an extra utility through its effects on the improved school quality. Therefore, those students with \( \eta>0 \) purchase more \( e_2 \) to take advantage
of the lowered "effective price" differential.

In other words, students compete to enter better schools since all school education is uniformly priced (at 0 nominal price) regardless of the quality differentials among schools. In this sense, school quality is a free gift bestowed upon the ε purchaser. Consequently, students have an incentive to exploit the quality differential to their maximum benefit. If, instead, the quality premium is properly and separately priced so that a higher quality education is purchasable only for a higher price, the competition to get into a better school would have not taken place. This implies that a part of the demand for private tutoring comes from the imperfect pricing of school quality differentials.

This raises the following question. Do we have similar competition to obtain a higher quality in the private tutoring market? Obviously not, at least in our model because our model does not allow any competition for quality to take place in the private education market\(^\text{(28)}\). Is this then a mere modeling artifact or more or less an accurate description of the reality? Probably the latter might be closer to the truth.

In the private tutoring market, competition to obtain a better quality education does not arise because there is a free competition among suppliers and demanders in terms of both price and quality. Firstly, competition among students to buy higher quality education will drive up the price of the higher quality good, until the price differential disappears. Therefore, all the quality differential in equilibrium will be reflected in the price, and consequently, competition for a higher quality will be transformed into a competition for a merely larger quantity. In this regard, the \( p \) term refers to the effective quality adjusted unit price of private education. Of course, this is an implicit assumption exogenously imposed on the model, not a conclusion derived within the model. Nonetheless, it appears to provide a quite plausible and reasonable

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\(^{28}\) Private tutoring is assumed to be all homogeneous in our model. Is it because there is no quality difference in private education?
interpretation to the results we have. Secondly, in the private education market, the homogeneity of the student body may be more easily secured because more individualized care by tutors is possible. In contrast, the nominal price in the public education market is uniform, and as a result, the real effective prices are all different. Furthermore, individually tailored education is more difficult in school education. This implies that in the school education market, there may exist some rigidity which hinders a smooth adjustment of the real effective price, even though it is not clear at this stage why and how such rigidity persists.

5-3. Comparative Statics and Properties of the Equilibrium

The second order conditions for the two interior solutions to be utility maximizing are

$$\begin{vmatrix} H_1 \end{vmatrix} = U_{xx}p^2 - 2pU_{xe}(\eta + 1) + U_{ee}(\eta + 1)^2 + U_e\eta' < 0, \quad (22)$$

for the first interior solution, and,

$$\begin{vmatrix} H_2 \end{vmatrix} = \begin{vmatrix} U_{ee} - 2pU_{xe} + p^2U_{xx} & 0 \\ 0 & \eta' \end{vmatrix} \quad (23)$$

$$= (U_{ee} - 2pU_{xe} + p^2U_{xx}) \eta' > 0$$

for the second interior solution. These conditions are met under the current assumptions. Note that we have only one control variable, $e_2$ ($= e_{21} = e_{22} = \bar{e}_2$), for the former, and two control variables, $e_2$ and $e_3$, for the latter. This means that both the amount of private tutoring and school quality are jointly determined in the former, whereas, in the latter, they are separately determined. With these at our hands, we can

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29) As usual, these second order conditions are merely conditions for the indifference curves to be convex toward origin.

30) Note that the off-diagonal elements in (23) are all zero, implying $e_{22}$ and $e_{21}$ are independent each other. For comparison, (22) can be rewritten as

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straightforwardly perform the following comparative statics.

1) \( y \) effect

\[
\frac{\partial e_2}{\partial y} = \frac{pU_{xx} - U_{xx}(\eta + 1)}{|H|_1} > 0
\]

\[
\frac{\partial e_{21}}{\partial y} = \frac{[pU_{xx} - U_{xx}]\eta'}{|H|_2} > 0
\]

\[
\frac{\partial e_{21}}{\partial y} = 0
\]

2) \( p \) effect

\[
\frac{\partial e_2}{\partial p} = e_2[pU_{xx}(\eta + 1) - pU_{xx}] + U_x \frac{|H|_1}{|H|_1} < 0
\]

\[
\frac{\partial e_{22}}{\partial p} = \frac{[e_2(pU_{xx} - pU_{xx}) + U_x] \eta'}{|H|_2} < 0
\]

\[
\frac{\partial e_{21}}{\partial p} = 0
\]

\[|H|_1 = \begin{vmatrix} U_{ee} - 2pU_{xx} + p^2U_{xx} & (U_{ee} - pU_{xx})\eta & 1 \\ (U_{ee} - pU_{xx})\eta & U_{ee}\eta^2 + U_{e}\eta' & -1 \end{vmatrix} \tag{22}'
\]

for \( e_{22}, e_{21} \) and for some constraint \( \kappa \) which assures \( e_{22} = e_{21} \). From this, it is evident that the condition (23) is obtained by imposing the condition on (22)' that \( \eta = 0 \) and dropping the condition \( e_{22} = e_{21} \).
3) $\bar{e}_1$ effect

$$\frac{\partial \bar{e}_2}{\partial \bar{e}_1} = \frac{pU_{\infty} - U_{\infty}(\eta+1)}{|H|_1} < 0$$

$$\frac{\partial e_2}{\partial \bar{e}_1} = \frac{pU_{\infty} - U_{\infty}}{|H|_1} \tilde{\eta}' < 0$$

$$\frac{\partial e_1}{\partial \bar{e}_1} = 0$$

4) $\alpha$ effect

$$\frac{\partial e_2}{\partial \alpha} = \frac{2\beta(a-q)[U_{\infty}(\eta+1) - pU_{\infty}]}{|H|_1} - \frac{2\beta U_1 \psi'}{|H|_1} = ?$$

$$\frac{\partial e_2}{\partial \alpha} = \frac{2\beta(a-q)[U_{\infty} - pU_{\infty}]}{|H|_1} \tilde{\eta}' = ?$$

$$\frac{\partial e_1}{\partial \alpha} = -\frac{2\beta \psi'}{\eta'} > 0$$

5) $\alpha$ effect

$$\frac{\partial e_2}{\partial \alpha} = \frac{\hat{\bar{e}}_1[pU_{\infty} - U_{\infty}(\eta+1)]}{|H|_1} - \frac{U_1 \hat{e}_1 \tilde{\eta}'}{|H|_1} = ?$$

$$\frac{\partial e_2}{\partial \alpha} = \frac{\hat{\bar{e}}_1[pU_{\infty} - U_{\infty}]}{|H|_1} \tilde{\eta}' < 0$$

$$\frac{\partial e_1}{\partial \alpha} = -\frac{\hat{\bar{e}}_1 \tilde{\eta}'}{\eta'} > 0$$
6) $\beta$ effect

$$\frac{\partial e_2}{\partial \beta} = \frac{(a-q)^2[U_{e_2}(\eta+1) - pU_{e_2}]}{|H_1|} - \frac{2(a-q)U_e}{|H_1|} = ?$$

$$\frac{\partial e_{2i}}{\partial \beta} = \frac{(a-q)^2}[U_{e_2} - pU_{e_2}] \eta' \geq 0$$

$$\frac{\partial e_{2i}}{\partial \beta} = -\frac{2(a-q) \psi}{\eta'} = ?$$

Some explanations about the results are in order. As long as the amount of private tutoring is concerned, the first two are the standard results under the assumption of normality of educational good in both interior solutions: the demand for private tutoring increases as the income rises, and decreases as the price of private tutoring increases. What is peculiar here is the equilibrium school quality chosen by students in the second interior solution, which is invariant to changes in income, price and even $\bar{e}_1$. In the second interior solution, students’ demand for school quality is saturated, and, therefore, is not affected by a marginal change in budget constraint$^{31})$.

Third, the demand for private tutoring decreases as $\bar{e}_1$ increases. As assumed in the model, $\bar{e}_1$ represents an exogenously-given school education. Thus, an increase in $\bar{e}_1$ stands for an improvement in the school education system. As more education is freely given in the school, the demand for the out-of-school education declines$^{32})$.

Starting from the fourth to the last, all the results above have two

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31) Of course, the boundary locus will be changed accordingly.
32) Strictly speaking, this requires the normality of $x$, which is true under the current assumption of $U_1<0$ and $U_2>0$. As the quantity of educational good, $e$, increases, a student wants to consume more of $x$, because the marginal utility of education diminishes.
terms in the first interior solution, one representing the demand for private tutoring that arises from student’s need to supplement his/her school education in a given quality school, and the other the demand for private tutoring for improving the quality level itself. The latter works through the induced change in the marginal effectiveness of private tutoring in improving the school quality of a student’s choice. In other words, the latter represents $U_e \frac{\partial \eta}{\partial z}$ where $\partial z$ stands for a parametric change of the model. Note that $\eta$ represents the marginal effectiveness of $e_2$ in increasing educational achievement via quality improvement. In the second interior solution, these two terms are separated, because once the school quality is set at the saturation level, the demand for private tutoring is solely determined by the former alone. In this case, the second term does not represents the demand for private tutoring, but only the change in the level of saturated quality.

The fourth one shows how students with differing ability behave with regard to their choice of private tutoring. As explained above, it has two components. Note, however, that the expressions in the first equation are basically the sum of the second and third equations, which shows the two separate effects in the second interior solution. We may, therefore, look at the two distinct effects in the second interior solution first, and then interpret the results as happening simultaneously in the first solution.

In the second interior solution, the demand for private tutoring crucially depends on the sign of $(a-q)$ gap. In the range of $a>q$, the demand for private tutoring increases as student’s ability index increases, implying that the higher the ability, the larger is the demand. In the range of $a<q$, however, the sign is opposite: the demand for private tutoring decreases as the ability index increases. In this range, students buy the less, the higher their ability is. Appearing a bit complicated, it in effect says that students who are far away from the quality level of their own school will buy more private tutoring, which is logically sensible. They do not get as much education as those who are close to the school
quality do, so they go out to private education market to make up for the deficiency.

The demand for a better school is, however, monotonically increasing in \( a \), implying that the greater the ability, the higher quality school he/she would want to go, and thus purchase the more private tutoring. This is also an intuitively appealing result. The underlying logic is that the higher the student ability, the smaller is the penalty that he/she has to pay for the mismatch when he/she advances to a higher quality school. Going back to the first interior solution where these two effects work together simultaneously, it implies that the higher-ability students would definitely buy more private tutoring because these two effects operate in the same direction. For the lower-ability students, however, the combined effects are uncertain, because the two effects move towards opposite directions each other, one for increasing the demand for private tutoring, the other decreasing.

The fifth one presents the effect of change in \( a \), the parameter representing the amount of school education one can get by attending a given level of quality school. In the second interior solution, the demand for private tutoring decreases as \( a \) increases. This is because an increase in \( a \) causes the amount of school education at a given quality school to rise. Therefore, demand for additional education in the private market decreases. The effect of an \( a \) increase on the quality improving demand for private tutoring is, however, positive. As \( a \) increases, the effectiveness of private tutoring in raising the total educational attainment through quality improvement is enhanced. This induces students to make more investment in private tutoring. The saturation level of school quality is also heightened because a student now can enjoy a higher quality education without incurring an additional penalty. Here, these two effects, therefore, operate toward two opposite directions for everybody, one for decreasing the demand, and the other increasing. When combined into one in the first interior solution, the net effect is, therefore, indeterminate. Those students who are already attending the highest possible quality
school at the saturation point, however, will buy definitely less private tutoring, because the latter effect is not in force for them\textsuperscript{33}).

Sixth, as far as the effect on private tutoring is concerned, the effect of $\beta$ change works in the opposite way to that of an $\alpha$ change. It first increases the demand for private tutoring for most of the students by reducing the school education one can get through the magnified penalty. Of course, this effect crucially depends on the size of discrepancy between the ability and quality. For those students who have established a perfect match, this effect is zero. Those students whose ability is far from the school quality, the penalty is proportionately higher as the gap widens, and, therefore, choose to buy proportionately more private tutoring. This suggests that the equalization measure (to be discussed more fully later) might have prompted more tutoring, not less as desired, because it inevitably increases the number of students who are away from the school quality in every school.

The demand for private tutoring for the purpose of entering a better school also hinges on the gap between quality and ability, but this time, on the direction of the disparity, not the size. For those students whose ability is higher than the school quality, improving the school quality through an additional investment in private tutoring is desirable. They used to attend a school whose quality is lower than their own ability because the quality is costly. Now that the mismatch penalty is higher, they have a stronger incentive to move up to a higher quality school even if it entails a additional cost. Therefore, high ability students purchase more tutoring. For those students whose ability is lower than

\textsuperscript{33}) It may sounds a bit counter-intuitive, at least on the first impression. An increase in $\alpha$ which will make $e_2$ more effective in raising the educational outcome has indeterminate effects on $e_2$? Of course, this reasoning has been taken care by our second term, the indirect effect. Somehow, one tends to forget the first direct effect: having a larger amount of school education will induce students to choose less of out-of-school education, its direct substitute. In fact, we tend to forget the fact that there are also other goods to consume, other than education, and an exogenous increase in school education will prompt one to cut the out-of-school education and instead to choose more of other goods.
the school quality, however, a further upgrading of school quality is the last thing they want to have. An increase in $\beta$ in this situation merely increases the penalty. As a result, it induces students to cut down their demand for school quality. Again, in the first interior solution, these two mutually conflicting forces must be summed, and therefore, the aggregate effect is indeterminate.

One interesting implication we can draw from the above results is that a school with a given quality cannot sort its students’ ability just by looking at their parents’ income, the crucial assumption that drives the main results in Epple and Romano (1998).

**Proposition 7:** There is no simple trade-off relationship between income and student ability at a given quality school. For some lower value of $a$, income decreases as $a$ increases, and then after passing a certain threshold value of $a$, $y$ increases as $a$ increases in the first interior solution. In the second interior solution, there exist no relationship at all between income and ability.

**Proof:** From 1) and 6) above, we have the following trade-off between $y$ and $a$ to have a given level of $q$ in the first interior solution.

$$\frac{dy}{da} \bigg|_{a_1-a_2=0} = \frac{2\beta(a-q)p[U_{yx}-U_{xy}(\eta+1)]+U_{xy}\phi'}{pU_{xx}-U_{xy}(\eta+1)} = ?$$  \hspace{1cm} (24)

In the second interior solution, this slope is not defined because the denominator is zero.

Note that, as explained earlier, a higher income family has a higher MRS for education, and as a result, a larger purchase of $e_2$. This is what Epple and Romano (1998) call the "single crossing property" in income, and holds, in general, when the commodity in question is a normal good. Unlike income, however, there is no simple, increasing or decreasing relationship between the student ability and the equilibrium amount of
private tutoring in the first interior solution. In the second interior solution, there does exist a clear cut relationship between quality and income, and between income and the amount of private tutoring. Unfortunately, however, the quality in this case has nothing to do with income, thus again generating the indeterminacy.

It is clear that this indeterminacy in both cases originates from the penalty effect we have introduced in our model. In the second interior solution, if we do not have the penalty for mismatch, we would not have the saturation point, either. In the first interior solution, the prime culprit for this inconclusiveness is the \((a-q)\) term in the numerator, which again comes from the penalty effect.

Graphically speaking, when \(a\) increases, \(U_o/U_s\) curve can either shift up or down depending on whether the student ability is higher or lower than the school quality at the equilibrium. If it is higher, i.e., \(a>q\), then the MRS curve shifts up as the student ability increases, and if it is lower, i.e., \(a<q\), then the curve shifts down.

A student whose ability is higher than his/her school quality finds it beneficial to improve the school quality further. So he/she is willing to give up other goods to buy one marginal unit of school quality, and this willingness increases as the student’s ability index gets higher because the marginal returns to quality improvement gets bigger. For a student who has already chosen a quality higher than his/her ability, however, a further increase in quality is not something desired for in itself unless a redeeming compensation is provided. The compensation, the

\[\text{34) It does not depend on the particular functional form of the penalty term. For example, altering the quadratic function into an absolute value function would still generate the same indeterminacy.}\]

\[\text{35) The second term in the numerator in (24) only shifts the threshold point without changing the major implication of the result.}\]

\[\text{36) This is most likely if he/she is born with a high ability.}\]

\[\text{37) This may be a bit confusing. We know there are three different ranges of quality a student with a given ability to choose: } q>q^*(a), q^*(a)>q>a, \text{ and } q<a. \text{ In the range } q>q^*(a), \text{ every student will choose } q^*(a), \text{ because any quality higher than } q^*(a) \text{ is a real pain. This is the case of the second interior solution. The first interior solution}\]
additional gain they can get from quality improvement, however, gets smaller as the ability index moves closer to the a=q point. As a result, students become less inclined to sacrifice other goods to have a higher quality as they get a higher ability, which causes the $U_w/U_x$ curve to shift down.

In the range where there is a negative trade-off between y and a, a very high value of y is associated with a very low value of a at a given level of q, and vice versa. Therefore, a given level of quality is purchased either by families with a high income but a low a, or families with a low y but a high a, or families in between the two, implying that the student body of a given quality school will be composed of the students who satisfy this particular trade-off relationship. On the individual family level, this implies that a family which does not put a high value on education would buy their kids a small amount of out-of-school education, or refuse to buy at all, unless their kid demonstrates an exceptionally high ability. On the other hand, the family which values education very highly will spend lavishly on private tutoring for their kids even when the kid shows mediocre ability\textsuperscript{38}).

For the reason already explained, however, this type of stratification of students is bound to be incomplete. In the first solution, some students whose ability is located close to the lower end purchase less and less quality as their ability increase toward a=q point (or, slightly lower than this). In order to induce them to purchase the same amount of quality, therefore, you have to give them a correspondingly higher income. This creates a positive relationship between ability and income in this range. In the second solution, the situation is worse, having no relationship at all \textsuperscript{39}).

\textsuperscript{38}) This may open the possibility for the schools to practice some sort of price discrimination of the type discussed in Epple and Roman (1998), i.e., offering a scholarship to some high ability students. But, again, the caveat referred to in footnote (15) applies here.

\textsuperscript{39}) Putting aside the saturation solution as a remote possibility, this suggests that there
If schools are interested in securing students with high intrinsic ability, this implies that schools will have difficulty in achieving their goal because, in the current setting, schools have no way of knowing the true ability of a student, even among those students who have successfully secured the school’s admission by demonstrating a high enough academic achievement, i.e., the amount of $e_{21}$ purchased. The high performance may be the result of 1) student’s intrinsic ability, 2) family’s high income, and information about $y$ is not that helpful. For an effective price discrimination, schools may have to require the students to submit additional information$^{40}$.

5–4. Effects of School Equalization Measure

In the early 1970s, the Korean government adopted a rather radical reformatory measures in secondary education system by banning all forms of school entrance examinations which had been in use ever since the adoption of the modern education system after WW II. The major goal of this measure was to reduce private tutoring, which was viewed “unreasonably excessive.” Before this measure was taken, each student had to pass an entrance examination which was administered by individual schools. There had been no zoning restrictions either. In that sense, there was a national level competition to enter a better school, and there had been a well-established hierarchical order amongst secondary schools. The authority perceived that the excessive demand for private tutoring had originated in large measure from this school-order, and that by eliminating this rank-order system, one could dampen or root out the passion for private tutoring. Under the new equalization measure, students were randomly allocated, basically by lottery, to schools within the newly

$^{40}$ will be a positive correlation between $y$ and $a$ in the lower end of $a$, and then a positive correlation in the upper end, which may be testable with empirical data.

$^{40}$ Of course, schools can be assured of the lowest ability, because $q^{-}$ is increasing in $a$. 

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created school districts in the neighborhood of the students' residence.

In a comparative sense, therefore, we can see that there exists a close similarity between the proposals aired these days in the US to improve its school system and what the Korean government did three decades ago to reduce the excessive competition, but in an opposite way. The so-called "choice" policies or proposals include inter- and intra-district open enrollment, formation of magnet and charter schools, and vouchers for private schools. Thus, the goals that the choice proposals aim to achieve can be restated as increased school choice through an elimination of territorial restrictions and a mitigation of the familial financial constraints, which are in a large extent diametrically opposite to what the Korean government had implemented: a new creation of school districts, an introduction of mandatory neighborhood schooling, and prohibition of competition among schools and students.

In order to analyze the effect of the school equalization measure, I assume in this section a more concrete form of school quality choice function. Specifically, I assume,

\[ q = k e_2, \]  

so that \( \Psi' = k \). In this formulation, \( k \) stands for the marginal return of private tutoring in enhancing the quality of school. The equalization measure can then be interpreted as setting the value of \( k \) at zero, or reducing it to a small value, rendering the private tutoring less potent or impotent at all in its power to raise the school quality. I take the equalization measure as marginally reducing the value of \( k \), and perform the following comparative statics \(^{41}\).

\(^{41}\) There are some reasons for not setting \( k=0 \). Obviously, it makes the analysis easier. In addition to this, the school equalization measure was in some sense incomplete. In the case of junior high school, only those schools located within the major metropolitan cities were subjected to this new ordinance, and schools in local provinces were left free to take either form of the admission methods: free competition or compulsory randomization. Furthermore, in tandem with this equalization measure, a handful of special schools were newly launched to
A differentiation of the first order condition of the first interior solution with respect to $k$ gives \(42\),

\[
\frac{\partial e_2}{\partial k} = - \frac{\xi_0 e_2 \left[ U_{\xi_0} (\eta + 1) - p U_{\xi_0} \right]}{|H|_1} - \frac{U_\xi \left[ (a \, \hat{e}_1 - 2 \beta) e_2 k + \xi \right]}{|H|_1} = \xi e_2 \frac{\partial e_2}{\partial \hat{e}_1} \quad - \quad \frac{U_\xi (\frac{\partial \xi}{\partial \hat{e}_1} q + \xi \xi)}{|H|_1}
\]

\[
= \xi e_2 \frac{\partial e_2}{\partial \hat{e}_1} \quad - \quad \xi e_2 \frac{U_\xi (\varepsilon_{\xi_0} \varepsilon_{\xi_0} + 1)}{|H|_1} = (-) \quad + \quad (?)
\]

where $\varepsilon_{\xi_0}$ is the elasticity of $\xi$ with respect to $q$ and $\varepsilon_{\xi_0}$ the elasticity of $\eta$ with respect to $k$.

In the range of the first interior solution, $\xi$ is positive. The first term, which gives the direct effect of the $k$ increase is negative. For a given $e_2$, an increase in $k$ brings about a higher $q$, via $q = k e_2$, which increases $e_1$. This reduces the demand for private tutoring. The second term is indeterminate because the elasticity of $\eta$ with respect to $k$ can be positive or negative. If it is negative, then the overall effect which combines both the first term and the second, is negative, implying that private tutoring will decrease as $k$ increases. If it is positive, the overall effect is ambiguous, and will depend on which one is bigger in absolute terms.

School equalization measure lowers the value of $k$, and does not increases it. Therefore, we have to reverse all the signs in the above

\footnote{As already explained above, the analysis for the second interior solution is almost similar to the first case, except that the two effects are now separated.}

accommodate the high ability students such as special schools for science and special schools for foreign languages. These schools were allowed to have entrance examination to select students, and later became very competitive. Still another reason is that the colleges and universities have never been equalized.
results. Under this reversal, the first term is positive, implying we will have more private tutoring as a result of the equalization measure. This is opposite to what is intended. The second term is either positive or negative. If it is also positive, that will be a fatal blow to the equalization measure because the overall effect is unabashedly opposite to what was aimed for. If it is negative, the overall effect is indeterminate. In short, the equalization measure may have been altogether detrimental to the reduction of private tutoring, the result so desperately hoped for, or may have been neutral or ineffective because of the countervailing forces at work. It may have been effective if, and only if, the indirect elasticity effect is so large that it completely cancels out the first direct effect and works more to the negative direction, which is a rather stringent requirement to satisfy.

The intuitive reason for this intriguing conclusion is not difficult to figure out. Let’s take the direct effect first, the demand for private tutoring to supplement school education. The channel through which students can attain a larger education within the school system by attending a high-quality school is now damaged by the equalization measure. As a result, students with a given ε₂ now have a smaller ε₁ to consume than before. Consequently, they have to resort to alternative means to fill up their unsatisfied educational demand, which is buying the same good in the private market. In short, people buy more private tutoring because they can not get what they want within their schools.

The second effect, i.e., the demand for private tutoring for the purpose of going to better schools, is a bit involved. A change in k causes η to vary, the marginal productivity of private tutoring in increasing the school education. Note that η=ξk, and when k decreases, ξ moves to the opposite direction because ξ is decreasing in q. 43) Thus the overall effect of a reduction in k on η depends on how much ξ increases in response to the k reduction. If ξ increases more than

43) This is the second order condition we have seen earlier. If it doesn’t hold, every student will try to go to the highest possible quality school, destroying the equilibrium.
proportionately, that is, if $\xi$ is elastic with respect to $k$, $n$ increases. This induces students to take even more private tutoring in response to the $k$ reduction. In this case, lowering $k$ will do more harm than good, as far as the volume of private tutoring is concerned. On the other hand, if $\xi$ increases less than proportionately, i.e., if $\xi$ is inelastic, a reduction in $k$ decreases $n$, and thus discourages private tutoring. In this case, the direct effect and indirect effect work in the conflicting manner, one to increase and the other to decrease the private tutoring, and the overall effect becomes uncertain.

Recall that $\xi$ stands for the marginal productivity of quality in increasing the amount of education a student can get within the school. In other words, it is an indicator about what significant difference it can make to attend a school whose quality is higher by one unit. We know that this is diminishing. The question is then how fast it decreases: More than proportionately as $q$ increases or less than proportionately? This is the elasticity question we have had above.

The elasticity of $\xi$ function depends on the shape of $\hat{e}_1(q)$ function and the values of $q$ and other parameters. More specifically, it can be shown that in order for $\varepsilon_{\hat{e}_1}$ to be inelastic, one must have

$$-\frac{\hat{e}_1''}{\hat{e}_1'} q < 1 - \frac{4\beta q}{a \hat{e}_1}. \quad (27)$$

This implies that for a given $\hat{e}_1(q)$ function, the smaller (larger) the $\beta (\alpha)$ is, the value of the right hand side of the above inequality will become larger, rendering it easier for the above condition to be satisfied.

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44) For a given set of parametric values, a logarithmic function of the type $e_1 = \log\ (1+q)$ yields the left hand side of the inequality to lie between 0 and 1, and a power function of the type $e_1 = 1-1/(q+1)$ between 0 and 2. Therefore, it is more likely that the elasticity condition holds under the latter function. For example, in the case of logarithmic function, if the value of $\beta$ goes to zero, $\varepsilon_{\hat{e}_1}$ will be always inelastic, thus making the sign of the second term positive.
There are at least two more things that we have to consider with regards to the equalization measure adopted by the Korean government. With the abolition of all forms of entrance examinations, the authority had to provide some means of allocating students amongst the schools. As mentioned earlier, the measure adopted was to randomize the allocation through a lottery. Many new school districts have been defined based on, among others, regional proximity, in such a way that each district contain multiple schools, and within each school district, students were randomly distributed among schools45).

The net effect was a huge increase in the variance of student ability distribution within each school. Before the equalization measure, students self-selected themselves according to their ability and income, making it possible for each school to hold a comparatively homogeneous ability group. Under the newly introduced equalization measure, they were forced to attend a school whose student ability distribution is not known, or, worse than that, where the distribution was manipulated to be almost the same as the distribution of the student population within a school district in every school. Within the current model, it means that the area of free disposability has been turned into no-disposability, thus increasing the range of the ability distribution of every school.

If we interpret this as an increase in $\beta$ in our school education function, it may have increased private tutoring further, at least among those whose ability is relatively high.

Lastly, the equalization measure has effectively destroyed the distribution of school quality itself and caused the distribution to

45) In reality, it was not purely random, though, because the authority consciously tried to achieve an equalization of school even in student ability distribution among schools, probably to perfect the spirit of equalization in every aspect of the matter or to ward off possible complaints from parents. It somehow ranked each student’s ability within a given school district, and then assigned the students to different schools in a descending order such that the ex post mean and variance of ability distribution in each school should be almost comparable across schools.
degenerated. First, the measure literally ordained the shutdown of several top-quality schools. Second, among those that survived, little incentives were there to continue to try to provide top-quality education to students. Third, even if they tried, it was difficult to achieve the desired outcome because the range of the distribution of student ability within a school was practically too wide to have effective teaching and learning\textsuperscript{46). A degeneration of school quality distribution in our model implies that a large number of students must have dropped out of school. As mentioned earlier, this does not necessarily mean that all the students in this range literally have stayed out of school. It might have been difficult for those students to actually quit coming to school under social and family pressure. In this regard, a testimony on the Chinese experiment which is similar to what Korea had, is suggestive about what would have happened: "What only became clear to me later,... was the collapse of real learning in the primary and secondary schools; in Unger's words, 'most of China's urban students simply stopped paying attention in class," \textsuperscript{47). In the context of our model, it means that a large number of students located near the end points of the ability spectrum must have been driven to go directly to the private tutoring market to satisfy their demand for education.}

All in all, it is impossible to draw any definite conclusion about the effects that the equalization measure may have created on the amount of private tutoring. Other than the last one, which is definitely positive, the signs of the first two effects are mixed. It is clear, though, that if we confine ourselves to the direct effects only, the measure must have been detrimental to what is desired for, because all those terms are decidedly positive. Furthermore, some of the available circumstantial evidences are also pointing to that overall direction \textsuperscript{48).}

\textsuperscript{46) There exists abundant testimonies to that effect.  
48) See Kim and Lee (2001) which shows that the private tutoring has been larger in the metropolitan cities than in medium and small cities, even after the effects of other variables are controlled.}
References


