Risk Aversion and Incentive to Abide By Legal Rules

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Abstract

Does a more risk-averse individual put greater effort to abide by legal rules than the less risk-averse? Or are the criminals really a race apart who is exceptionally risk prone? This paper examines the relationship between the law-abiding effort of an individual and his degree of risk aversion. It is shown that a monotone positive relationship between risk aversion and law-abiding cost holds only when the probability of legal sanction is exogenously given. Once the probability is endogenously chosen by the potential perpetrator, this simple relation no longer holds. In short, a fairly risk-averse individual can still commit a crime.

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**Risk Aversion and Incentive to Abide by Legal Rules**

If legal rules are enforced perfectly, in the sense that there is no vagueness in the proclaimed statutes, no mistakes, no measurement error, no delay in implementation, etc., people will faithfully abide by every rule. In reality, however, laws are always enforced imperfectly. Not every wrongdoer is reported to the authority, and some are left at large. Courts make mistakes in implementing the existing laws. Fact finding processes are marred with measurement error, inconclusive evidences.

In this situation, the rewards and punishments that a law-abider is effectively faced with are at best random. Some may be punished unnecessarily harshly while others are exploiting the loopholes and imperfections of the legal system. Faced with this kind of legal uncertainty, would a more risk-averse individual put greater effort to abide by the legal rules and regulations?

At first blush, it appears that a risk-averse individual hates uncertainty and therefore would try more strenuously to comply with legal requirements. He would buy more of certainty by making a greater amount of investment into the law-abiding activity. In line of this kind of belief, some sociologists or criminologists even suspect that criminals are people of special breed who are exceptionally prone to risk, and therefore, would not respond in an expected way to the system of incentives designed primarily for ordinary risk-averse individuals.

Would the law-abiding incentives have indeed a monotone relationship with the degree of risk aversion of the individual? Are the criminals really special in their attitudes toward risk? This is the question I would like to address in this paper. Using a simple expected utility maximization model, I examine the relationship between the
cost one is willing to incur to abide by rules and his attitude toward risk. It will be shown that the relationship is not always unambiguous, but depends on the particular legal environment that a decision maker is surrounded. In particular, if the probability of legal sanction on wrongdoings is exogenously given, an individual will exert a greater effort as his risk aversion rises. However, once the probability of sanction is endogenously chosen by the individual, this simple relationship no longer sustains. There arises a possibility that a reverse relationship holds.

Theoretically what is crucial in this context is whether the additional investment improves or worsens the bad state wealth: if it improves, a more risk-averse individual always chooses to take a greater cost than the less risk-averse. This is because the additional investment induces the resulting wealth distribution to stochastically dominate the original distribution in the sense of mean preserving contraction. If it fails to improve the bad state wealth, the stochastic dominance does not hold, and consequently, the relationship turns into ambiguity.

The paper is organized as follows. In the next section, I lay out a simple model which can explain individual’s law abiding behavior under various legal environments. In section 2, comparative static analysis is performed to see the direction of control variable in response to the increased risk aversion. Section 3 concludes.

I. The Basic Model

A person has to take some precautionary action x in order to reduce the harm he inflicts upon others l(x)>0, l’(x)<0. The precaution is, however, costly to the actor, c(x), c’(x)>0. The social objective is then to minimize the sum of the cost of precaution and
the loss\(^1\), \(\text{Min}_x d(x) = c(x) + l(x)\) the optimum of which is achieved when \(d'(x^*) = c'(x^*) + l'(x^*) = 0\). That is, the caretaker's marginal cost should be weighed against the marginal benefit, the reduced loss due to his additional care. In the following, we assume that \(d(x)\) is a strictly convex so that there is a unique interior solution. This implies that \(d' = c' + l'\) is monotonically increasing, with the sign single-switching from negative to positive at \(x^*\).

If there is no legal sanction against the harm inflicted upon others, people would take no care \((x = 0)\), completely disregarding the loss they cause to others. It inevitably leads to social inefficiency. To correct this suboptimal incentive, the society usually imposes some legal duty on people. For example, a society may set up a compensation rule which requires the wrongdoer to pay the damages, \(l(x)\) he inflicts upon others. This will make the negative externalities internalized.\(^2\)

Asking every actor to pay the negative consequences of his action to others requires a continuous monitoring of each individual's behavior, which is no doubt costly, often prohibitively. A less costly way is to set up a legal standard, \(\bar{x}\), and hold liable only those whose care level falls short of the required minimum.

Let \(r(x)\) be the liability exemption function such that \(r(x) = 1\) when \(x \geq \bar{x}\), and \(r(x) = 0\) when \(x < \bar{x}\). Assuming an individual has a utility function \(u(.)\), this liability assignment rule gives rise to the following maximization problem for a potential actor,

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\(^1\) For simplicity, I assume that there is nothing that others can do to reduce the loss. This is, therefore, the case of unilateral care.

\(^2\) In fact, this is what a market does when the property rights on \(x\) is well established. In this case, \(l(x)\) function takes the form, \(l(x) = bx\), where \(b\) stands for the market price of \(x\). The actor should compensate the value that other people is made to forgo due to actor's action.
\[ \max_x (1-r(x)) u(w-c(x)-l(x)) + r(x) u(w-c(x)) \]
\[ = (1-r(x)) u(w-d(x)) + r(x) u(w-c(x)) \]

(1)

where \( w \) is the initial wealth. It is assumed \( u' > 0 \), and \( u'' = 0 \). Therefore, all the results below apply both to those who are risk averse as well as risk neutral\(^3\).

Apparently, there exists a discontinuity at \( \bar{x} \) in individual's payoff function and this jump will make people faithfully abide by the rule. For every individual, taking precaution more than \( \bar{x} \) is certainly not profitable. Taking less, however, will cause an abrupt reduction in his net surplus. Thus, exact compliance with the required legal standard will be the outcome. This implies that if the legal standard is set at the socially optimal level, \( x^* \), the social efficiency is achieved.

Now we introduce three different types of legal uncertainty.

(1) The judicial authority does not always succeed in bringing all the wrongdoers to justice. A certain fraction, \( p \), of them (\( 0 < p < 1 \)) will still be at large. In the literature, this type of uncertainty is often called an "enforcement uncertainty," and I will call it "exogenous enforcement uncertainty" to differentiate it from the second type uncertainty below.

(2) The wrongdoer makes a positive investment, \( x \), to lower the probability of his being detected and subjected to the legal sanction, i.e., \( p(x) \) with \( p' > 0 \). Here, it is assumed that his investment has no effect on the loss size. His only concern is to run

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\(^3\) If the individual is risk-neutral, the maximization problem is reduced to the following minimization problem,
\[ \min c(x) + r(x) l(x), \]
which often appears in the negligence rule literature (Cooter and Ullen, 2000).
away and thus avoid the fixed legal sanction. We will call it “endogenous enforcement uncertainty,” because the enforcement probability is controlled by the wrongdoer himself.

(3) The same as in (2) above with one modification that the loss size is now a function of $x$ such that $l'(x)<0$. Therefore, the wrongdoer’s investment reduces both the probability of his being sanctioned and the damages he has to pay. It can happen when the judicial authority makes an error in measuring the actual level of care or commits mistakes in applying the legal standard. Following Cooter and Ullen (1985), we will call the uncertainty caused by this type of legal errors as "evidentiary uncertainty."

Given the classification, one can immediately notice the parallels to the existing insurance literature: the first case corresponds to the self-insurance in which the loss size is controlled by the actor with the probability of loss being exogenously fixed. The second corresponds to the self-protection where the probability of loss is controlled by the actor with a given size of loss. Finally the third case parallels the insurance-cum-protection in which both the probability of loss and the size of loss are simultaneously controlled by the actor. Correspondingly much of what follows is an application of the previous results in the insurance literature to a law-and-economics setting.

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4 If the court observes only $x+e$, not $x$ directly, where $e$ is the measurement error, a person will be exonerated from the liability only if $x+e \geq x^*$. Assuming that $e$ has density function $f(x)$, cumulative distribution function $F(x)$, we have $p=\text{prob}(x+e \geq x^*) = \text{prob}(e \geq x^*-x) = 1-F(x^*-x)$, so that $p'(x)=f>0$. To be precise, we have to assume some kind of additional uncertainty in the function of $l(x)$. Otherwise, the authority may be able to infer $x$ from the observation of $l(x)$ even if it has difficulty in directly measuring $x$ itself. If the courts commit mistakes in applying the legal standard, sometimes handling more stringently, other times leniently, we will have a theoretically equivalent formulation.

5 For self-insurance and self-protection model, see Becker and Ehrlich (1972), Dionne and Eckhoudt (1985), Briys and Schlesinger (1990), McGuire et al. (1991), and for self-insurance-cum-protection, see Lee (1998) and Julien et al. (1999).
In the following, I analyze how the actor’s optimal choice of \( x \) is affected by an increase in his risk averseness under the various uncertainty settings. It will be shown that under the exogenous enforcement uncertainty, a more risk-averse individual will always invest more in care-taking activity than the less risk-averse. However, under either the endogenous enforcement uncertainty or the evidentiary uncertainty, a more risk-averse individual will not necessarily invest more than the less risk averse individual. In particular, it will be shown that the sufficient condition for the more risk-averse individual to invest more is that \( l'(x)+c'(x)=0 \) or \( p \) is sufficiently small.

II. Comparative Statics

1. Under Exogenous Enforcement Uncertainty

The individual objective function under exogenous enforcement uncertainty is

\[
\max_x U(x) = (1-p)u(w_1) + p u(w_2)
\]

where \( w_1 = w - d(x) \) and \( w_2 = w - c(x) \), \( w_1 < w_2 \).

The FOC is

\[
U'(x_{ex}^u) = -(1-p)u'(w_1)d' - pu'(w_2)c' = 0,
\]

Where \( x_{ex}^u \) is the solution value of \( x \) (the subscript \( ex \) stands for the exogenous enforcement and the superscript \( u \) stands for the utility function \( u(\cdot) \). Assuming the SOC is satisfied, the following is immediate.
**Proposition 1:** Under the enforcement uncertainty,

(i) People will always under-comply;

(ii) A more risk averse individual takes more precaution than the less risk averse individual.

**Proof:** (i) Rewrite (3) as

\[-(1-p)u_1' d' = p u_2' c',\]

(4)

where $u_1'=u'(w_1)$ and $u_2'=u'(w_2)$. The right hand side is positive, which requires $d'<0$ on the left hand side. This implies that the individual optimum is always smaller than the social optimum, i.e., $x_{ex} < x^*$. 

*Figure 1* Equilibrium Care under Exogenous Enforcement Uncertainty
Using the fact that \(\frac{u_1'}{u_2'}<1\) when \(u''<0\), we can further show that

\[
\begin{align*}
    c'(x) &> -(1-p)l'(x) \quad \text{if risk averse} \\
    c'(x) &> -(1-p)l'(x) \quad \text{if risk neutral} \\
    c'(x) &< -(1-p)l'(x) \quad \text{if risk loving}^6.
\end{align*}
\]

Figure 1 shows the relationship.

(ii) Following Pratt(1964), let's take a utility function \(v(.)\) which is a concave transformation of \(u(.)\). That is, \(v=k[u(.)]\), \(k'>0\), \(k''<0\). Then \(v\) is more risk averse than \(u\). The corresponding FOC for \(v\) is

\[
V'(x^v) = -(1-p)v_1' d' - p v_2' c' = - (1-p) k'(u_1) u_1' d' - p k'(u_2) u_2' c' = 0^7. \tag{5}
\]

Evaluating (5) at \(x^v\), and making use of (4), we have

\[
V'(x^v) = [k'(u_1) - k'(u_2)] p u_2' c' > 0, \tag{6}
\]

which implies \(x^v > x^u\). The inequality in (6) comes from the fact that \(k(.)\) is concave and \(u_1 < u_2\).

Proposition 1, (i) and (ii) are the standard results obtained in a different context as well. As for (i) for example, the optimum output under price uncertainty is smaller than that under certainty (Sandmo, 1977). As for (ii), the more risk averse, the more is

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6 This implies that if the individual is infinitely risk-averse, the social optimum solution will be attained.
7 The subscript \(n\) and superscript \(v\) represent for enforcement uncertainty under utility function \(v\).
spent on self-insurance. This fact can be readily checked by its implied single-crossing property of the distribution functions (Jewitt, 1991). Before the expenditure, the cumulative distribution function F has \(F=0\) for \(w<w_1\), \(F=p\) for \(w_1\leq w<w_2\), \(F=1\) for \(w\geq w_2\). After the expenditure, the new distribution function \(G\) has \(G=0\) for \(w<w_1'\), \(G=p\) for \(w_1'\leq w<w_2'\), \(G=1\) for \(w\geq w_2'\). Therefore, the difference is \(G-F=-p<0\) for \(w_1\leq w\leq w_1'\), \(G-F=0\) for \(w_1'<w<w_2'\), \(G-F=1-p>0\) for \(w_2'<w\leq w_2\), which constitutes the single-crossing property. In short, an increase in the expenditure on care taking activity reduces the good state wealth from \(w_2\) to \(w_2'\), \((w_2'<w_2)\), but increases the bad state wealth from \(w_1\) to \(w_1'\), \((w_1'<w_1)\) because of \(d'<0\). Thus, it transfers wealth from good state to bad state. Consequently the difference between the two distribution functions, \(GF\), switches its sign from negative to positive.

2. **Endogenous Enforcement Uncertainty**

In this case, the actor makes an investment \(x\) in order to avoid the legal sanction of \(I\). Thus he has the following objective function:

\[
Max \quad U(x) = [1-p(x)]u(w_1) + p(x)u(w_2), \quad (7)
\]

with \(l(x)\) now turned into just a fixed \(l\). The FOC is

\[
U'(x_{en}^u) = p'(u_2-u_1) - [(1-p)u_1' + p u_2']c' = 0. \quad (8)
\]

In this case, both the bad and good state outcomes are shifted leftward, but the
probability of the bad state is lowered and the probability of the good state is improved.

Is this expenditure an insurance or a gamble? Dionne and Eeckhoudt (1985), McGuire et al. (1991) and Jullien et al.(1999) have the following.

**Proposition 2:** (i) A more risk-averse individual does not necessarily invest more in loss-preventing activity.

(ii) There exists some threshold probability \( p_u \) such that a more risk-averse individual invests more on loss-preventing activity than the less risk-averse when \( p > p_u \).

**Proof:** Following the same procedure as in the proof of Proposition 1 (ii), we have

\[
\phi = p'[k_2 - k_1] - [(1-p) k_1' u_1' + p k_2' u_2' ] c' = p'[k_3' (u_2 - u_1)] - [(1-p) k_1' u_1' + p k_2' u_2' ] c' = (1-p) (k_3' - k_1') u_1' c' + p (k_3' - k_2') u_2' c'. \tag{9}
\]

where \( k_i = k(u_i) \) and \( u_i = \alpha u_1 + (1-\alpha) u_2 \) for some \( \alpha \in (0,1) \). Since \( k(\cdot) \) is concave, \( k_3' - k_1' < 0 \) and \( k_3' - k_2' > 0 \). So we cannot sign the expression.

(iii) Note that \( \phi \) is increasing in \( p \). Also, \( \phi \) is negative when \( p = 0 \) and positive when \( p = 1 \).

Therefore, there exist \( p_u \) such that \( \phi > 0 \) if \( p > p_u \).

Note that the cumulative distributions, \( F \) and \( G \), cross twice, first at \( w_1 \) and secondly at \( w_2' \).

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\( ^8 \) Since \( p \) is a function of \( x \), this threshold value \( p_u \) is endogenously determined not exogenously given. Furthermore, there is a possibility that there exist multiple \( p_u \)'s.
3. Evidentiary Uncertainty

Now, let's turn to the case of the evidentiary uncertainty. We have the following objective function and the FOC condition.

\[
\max_x U(x) = [1-p(x)] u(w_1) + p(x) u(w_2),
\]

(10)

with \( l(x) \) now recovered, and

\[
U(x_{ev}^{l_0}) = p' (u_2-u_1) - [(1-p) u_1' d' + p u_2' c'] = 0.
\]

(11)

We can establish the following.

**Proposition 3:** Under evidentiary uncertainty,

(i) The equilibrium care level will not necessarily be smaller than the social optimum. It can be greater as well.

(ii) More risk-averse individuals will not necessarily take more precaution than the less risk averse.

**Proof:** (i) Evaluation of (10) at \( \hat{x} \) causes the first inside the bracket to drop out (\( d'=0 \)) and we are left only with

\[
p' (u_2-u_1) - p u_2' c',
\]

(12)

whose sign cannot be determined *a priori*. Rearranging, we have,
\[ \frac{ph_2}{x^2} \left[ \frac{ph_2}{x} \frac{x_2 - u_1}{u_2} - u_2 \frac{x^*}{u_2} \right] = \frac{ph_2}{x^2} \left[ \eta_{p,x}(1 - \frac{u_1}{u_2}) - \eta_{u,x} \right], \]  

(13)

where \( \eta_{y,z} \) denotes the elasticity of \( y \) with respect to \( z \). Note that \((1 - u_1/u_2)\) represents the relative gain one can enjoy by dodging the legal sanction. Therefore, the expression says that whether one takes more care than the social optimum hinges on the relative gain and two elasticity measures; one the elasticity of probability change due to one additional care and the other the elasticity of corresponding utility change, evaluated at \( x^* \). Simply put, if the run-away gain is easier to grab than the care cost to pay, he will over-comply. Otherwise, he will under-comply.

Another way of looking the same problem from a slightly different perspective is to write the FOC given in (11) as follows,

\[ p' (u_2 - u_1) - (1-p)u_1' d' = p u_2' c' \]  

(14)

as we did in (4). The right hand side is positive, which forces the left hand side to be also positive. But now this condition does not require \( d' \) to be negative any longer because the first term is positive, and as a result, the \( d' \) term has a freedom to take some positive as well as negative values.

The expression can be transformed into still another form which shed some light on the relationship between the relative risk aversion and the optimal compliance. Rewriting, we have
\[
p u_2' \left[ \frac{p' u_3'}{p u_2'} l' - c' \right],
\]

where \( u_3 = u(w_3) \), \( w_3 = \alpha w_1 + (1-\alpha)w_2 \) for some \( \alpha \in (0,1) \) and \( l' = l(x^*)' \). Note that \( u_3'/u_2' > 1 \) if \( u'' < 0 \), \( u_3'/u_2' = 1 \) if \( u'' = 0 \), and \( u_3'/u_2' < 1 \) if \( u'' > 0 \). Therefore, if a risk-loving individual chooses to take a socially optimal level of care, every risk-neutral or risk-averse individual will take a greater level of care than the social optimum. That is, it gives an example in which a more risk-averse individual takes a greater level of care than the social optimum. However, it should be noted that the comparison is confined only to the social optimum point where \( d'(x^*) = 0 \). In the following, it will be shown that this result cannot be generalized to other equilibrium values, especially so when \( x_{ev} < x^* \).

(ii) Using the concave transformation of \( u \), and rearranging, we have,

\[
\psi = (1-p) (k_3' - k_1') u_1' d' + p (k_3' - k_2') u_2' c'.
\]

Note that the only difference between this and (9) is \( d' \) term which has replaced \( c' \). But this small change has a great significance because \( c' \) is always positive but \( d' \) can now be positive or negative.

Let's see first the case where \( d' \leq 0 \). When \( d' \leq 0 \), the whole expression is positive, implying that more risk averse individuals take more precaution than the less risk averse. This is what Lee (1998) and Jullien and et al. (1990) called the "sufficient condition" for the more risk-averse individual to take more care than the less risk-averse individual.

When \( d' > 0 \) instead, the first term is negative and the second term is positive, and as

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9 The relationship that \( u_2 - u_1 = u_3' (w_2 - w_1) = u_3' l(x) \) has been used in derivation of above result.
a result, the overall sign of (16) is indeterminate.

Note that when $d' \leq 0$, an increased precaution reduces the dispersion of the wealth in the sense of the mean-preserving contraction. As a natural consequence, the more risk averse individuals invest more as the resulting distribution stochastically dominates the initial distribution in the second degree. Checking the shift of the cumulative distribution functions, one can easily confirm that the single crossing property does not indeed depend on the sign of $p'$ as long as $d \leq 0$. What is important is whether the bad outcome is improved or not by the investment, not whether the probability of bad outcome is lowered or not.

On the other hand, in the range where $l' > 0$, the stochastic dominance is no longer in force, and therefore, the investment decision depends on the trade-off between the improving probability and the worsening distribution of wealth. In this case, the more risk averse individuals invest more only when the probability of good outcome is high enough, as has been pointed out Mcquire et al. (1991), Lee (1998) and Julien (1999).

III. Conclusion

In this paper, the relationship between incentives to abide by rules and risk aversion under imperfect law enforcement has been examined. Contrary to the common belief that more risk-averse individual would pay greater attention to comply the legal rules, it has been shown that no such simple relationship holds under theoretical scrutiny. The incentives to abide by rules depend on the circumstances under which the decision maker is placed: under the exogenous enforcement uncertainty, it is true that the level of care increases as the actor’s risk aversion rises. Under the endogenous enforcement
uncertainty or evidentiary uncertainty, the decision maker’s investment does not necessarily increases as his risk aversion increases.

What is important in determining the direction of changes in control variable is whether the investment increases or decreases the wealth of the bad state, given the good state income is fixed at the initial wealth minus the investment cost: if it improves the bad state wealth, the decision maker increases his investment as the additional investment creates a new wealth distribution which stochastically dominates the original distribution in the sense of mean preserving contraction. In this case, the effects of the increased investment on the probability of loss are irrelevant in determining the sign of the comparative static results.

If the additional investment does not improve the bad state wealth, the effect of the increased risk aversion on decision maker’s investment is indeterminate. It is affected by the relative size of loss and changes in the probability of loss induced by the investment.

The model examined here can be extended in at least two directions. One is to introduce randomness in the initial wealth. As is known in some related literature, it is expected that the introduction of the additional uninsurable background risk would make it more difficult to have a clear-cut relation between the control variable and the increased risk aversion. This would be true even if we assume the stochastic independence between the initial wealth and the loss distributions.

The second possible extension is to examine the general characteristics of comparative static results under various assumptions about the utility functions, such as CARA, DARA, etc. Since the cost of care must be incurred in every state, regardless of the outcomes of the random variable, the changes in the degree of risk aversion induced
by the wealth change would certainly have perceivable effect on this risk taking behavior. This is left for a future research.
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