Factor Content of Trade: 
Nonhomothetic Preferences and "Missing Trade"

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Abstract

Recent empirical work has documented that the measured factor content of trade is too small compared to the theoretical prediction: missing trade. This misprediction may arise from the strong assumption that each country consumes services of factors in proportion to its share of world income. By incorporating nonhomothetic preferences into the model, this paper provides a simple resolution for the puzzle. The traditional Heckscher-Ohlin-Vaněk theorem consistently overpredicts factor content of trade compared to the model with nonhomothetic preferences. This result holds to the extent that income-elastic factors are on average abundant in rich countries and income-inelastic factors in poor countries.

Key words: Factor content of trade; Heckscher-Ohlin-Vaněk theorem; Income elasticity; Missing trade; Nonhomothetic preferences

JEL Classification: F11

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1. Introduction

During the past half century, trade economists have produced an extensive volume of research on the Heckscher-Ohlin-Vaněk (HOV) factor content of trade. The HOV theorem states that a country's factor content of trade equals the factor endowment of the country minus its share of world factor consumption. Given factor endowment information, the theory easily predicts whether a country will be a net exporter of services of a certain factor or a net importer of services of the factor and the volume of the factor content of trade. In the two-factor case, the theorem predicts that a capital-abundant country would be a net exporter of capital services and net importer of labor services.

Initiated by the so-called Leontief (1953) Paradox, most empirical studies of the Heckscher-Ohlin (HO) model have focused on testing predictions for the factor content of trade. However, its prediction had been badly rejected by data over and over again until Trefler (1993) came up with an empirical result that ratified Leontief's claim that the U.S. was actually a labor-abundant country when factors are measured in productivity-equivalent, efficiency units. Trefler (1995) went further to document that the measured factor content of trade is too small compared to the theoretical prediction. He labeled this serious misprediction of factor content of trade as "missing trade". He then examined alternative hypotheses and concluded that the model that allows for technology differences across countries and Armington home bias in consumption performed best. At that point, the HOV theory seemed finally to achieve empirical success. Trefler’s findings, however, turned out to be much weaker than originally claimed and perceived by many trade economists. Gabaix (1997) took the theory to a “genuine” test–his own term–using Trefler’s data. In order to adjust for cross-country technology differences, Gabaix employed factor rentals

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1 Among those empirical studies, Bowen et al. (1987) considered three alternative hypotheses, and rejected the HOV theorem in favor of models with weaker assumptions such as measurement errors, and technological differences.
2 Trefler (1993)’s result is distinct from Leamer (1980)’. Leamer showed that the U.S. was more abundant in capital than in labor in 1947 when factor contents are calculated allowing for the U.S. external surplus.
and per capita GDP as alternative measures of productivity. With these alternative productivity measures, Trefler's preferred model was rejected rather strongly.

Although Trefler’s result that seemed to support HOV was relatively undermined, the mystery of the missing trade, one of the novel contributions of his 1995 paper, was confirmed by Gabaix as well. The missing trade puzzle presents further evidence for the empirical inefficacy of the factor proportions model. More recently, however, Davis and Weinstein (2001) showed that it was possible to account for a significant portion of global factor trade by estimating parameters directly from relevant technology matrices of ten OECD countries instead of using the U.S. technology matrix as the basis for the other countries’ technologies. Their empirical results are so far the most successful evidence for HOV.

This paper attempts to provide another explanation for the puzzle of missing trade, in particular from the demand side. Almost all the previous empirical studies in this literature either modified the demand side to improve their test results or emphasized that some sort of demand-side modification must be made in order to put an end to its empirical misery. Nevertheless, we have not seen—at least to my best knowledge—a theoretical model that formally responds to this empirical outcry for developing a general equilibrium trade model in factor space with systematic demand-side modification of HOV. ³

We postulate that the missing trade puzzle is in part due to the serious misprediction, which may arise from the assumption of HOV that each country consumes services of factors in proportion to its share of world income. A simple way to resolve this problem is to replace this assumption with nonhomothetic preferences. For example, if the consumption of capital services is income-elastic and that of labor services is income-inelastic in the two-factor case, then capital-

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³ Marcusen (1986) briefly showed that with nonhomothetic demand, the volume of interindustry trade of goods could be reduced relative to the case of homothetic demand under certain assumptions for factor intensity and income elasticity. The main focus of his paper was, however, to explain both the volume and the direction of trade, encompassing both interindustry trade and intraindustry trade.
abundant, rich countries will consume capital services more than in proportion to their income shares and labor services less than proportionately. Likewise, poor countries will consume capital services disproportionately less and labor services disproportionately more. Consequently, the predicted factor content of trade between rich countries and poor countries will be reduced.

This paper incorporates nonhomothetic preferences into a general equilibrium trade model to provide an explanation, on the basis of per capita income differences, for the puzzle of the missing trade, or, more precisely, the missing factor content of trade. The insight that nonhomothetic preferences might have implications for the factor content of trade is hardly new. It was indeed intuitively well-sketched by Markusen (1986) and Deardorff (1998) among others. However, there has been no attempt to develop this idea formally and explicitly in factor space, which will have direct implications for the decades-long empirical ineffectiveness of HOV. In this paper, we first develop a simple $2 \times 2 \times 2$ model with nonhomothetic preferences for two factors, two goods, and two countries. We show in this context that the HOV theorem over-predicts the factor content of trade under certain conditions. This is illustrated in a simple diagram, with which the missing trade can be visually explained. The analysis then extends to a generalized model with arbitrary numbers of goods, factors, and countries, where income elasticity of consumption of factor services is defined by combining factor intensity of goods production and income elasticity of goods consumption. It is explicitly shown that a factor is income-elastic in consumption if it is on average used relatively intensively in production of goods that are income-elastic and unintensively in goods that are income-inelastic.

This paper further examines the relationship between per capita income, income elasticity of factor consumption, and the factor content of consumption to show that relatively rich countries must on average consume services of income-elastic factors more than proportionately while consuming services of income-inelastic factors less than proportionately. An indicator is devised to directly compare the volume of factor content of trade predicted by nonhomothetic preferences with the trade volume predicted by HOV. Direct comparison of this indicator generates the result
that the conventional HOV model not only predicts larger volume of factor content of trade for any country, but also for the world as a whole than does the model with nonhomothetic preferences. Therefore, the missing trade can in part be attributed to the excessive prediction of the HOV model. It is worth noting that the general result holds to the extent that income-elastic factors are on average abundant in rich countries and income-inelastic factors are on average abundant in poor countries. Obviously, whether this condition holds true is an empirical question. The whole literature of this research avenue has taught us that assuming homothetic preferences is not innocuous, especially in empirical studies. This paper provides a general equilibrium framework that can resolve the problem. Furthermore, since the framework is defined in factor space, it can be relatively easily adopted for an empirical study in this literature.

The paper is organized as follows. Section 2 develops a simple model that incorporates nonhomothetic preferences into the \(2 \times 2 \times 2\) HO model. Section 3 presents an integrated world equilibrium, in which we analyze how the factor content of trade can be predicted differently contingent on a simple difference in preference assumptions. In Section 4, we employ a more general form of nonhomothetic preferences, a linear expenditure system (LES), and illustrate loci of factor content of consumption for various preferences. The slope analysis of the loci demonstrates that the model with more general nonhomothetic preferences also confirms the result obtained in the simple model with an extreme form of nonhomothetic preferences. Section 5 investigates the link between income elasticity and the factor content of consumption in a generalized setup of many goods, factors and countries. In Section 6, an indicator for the volume of the factor content of trade is devised in order to provide a direct explanation for the missing trade. Section 7 concludes.
2. A Simple Model

Consider a traditional $2 \times 2 \times 2$ Heckscher-Ohlin model with nonhomothetic preferences. Let us denote two homogenous goods by $X$ and $Y$, two factors by $K$ and $L$. $K$ is capital stock and $L$ is labor force that is equal to population of the country. Let us name one country Home and the other Foreign. An asterisk will denote variables for Foreign. Suppose that a consumer $i$ has the following quasi-linear (QL) utility function:

$$U_i = x_i + u(y_i),$$

where we assume that $u(\cdot)$ is a strictly concave function. We will use $Y$ as the numéraire, $\phi_i$ as consumer $i$’s income in terms of the numéraire good $Y$, and $p$ as the price of good $X$.

From the above utility maximization problem an indirect utility function can be written as:

$$v_i(p, \phi_i) = \max u(y_i) + x_i$$

s.t. $px_i + y_i = \phi_i$

$$x_i \geq 0, y_i \geq 0$$

Applying the Kuhn-Tucker theorem, we can find two classes of solutions. First, let us consider the case where $x_i = 0$ and $y_i > 0$. Then the first-order conditions take the form

$$u'(y_i) \left( = \frac{du(y_i)}{dy_i} \right) = \lambda$$

$$y_i = \phi_i,$$

where $\lambda$ is the Lagrange multiplier. In this case, the marginal utility of $Y$ consumption must be equal to the marginal utility of income and all the income must be spent on good $Y$. Let us now consider the case where $x_i > 0$ and $y_i > 0$. The first-order conditions are

$$u'(y_i) = \lambda$$

$$1 = \lambda p$$

$$px_i + y_i = \phi_i.$$ 

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4 With the quasi-linear utility function of this kind, the usual notation is to set the $X$ good as the numéraire. To be consistent with notations used in later sections, we set $Y$ as the numéraire instead.

5 The quasi-linear utility maximization analysis draws heavily from Varian (1992).
Since \( u(\cdot) \) is strictly concave, the marginal utility of the consumption of good \( Y \) must be declining. We can find a certain income level, say \( \phi_o \), where \( u'(y_i) = \frac{1}{p} \). Together with this information, the first-order conditions suggest that for income greater than \( \phi_o \), the consumer’s marginal utility of income from consuming good \( Y \) is less than \( \frac{1}{p} \) while she can always get the marginal utility of income equal to \( \frac{1}{p} \) by consuming good \( X \). Hence her demand for \( X \) and \( Y \) will be

i) if \( \phi_i \leq \phi_o \), \( y_i = \phi_i \) and \( x_i = 0 \)

ii) if \( \phi_i > \phi_o \), \( y_i = \phi_o \) and \( x_i = \frac{\phi_i - \phi_o}{p} \).

If income is small enough as in case i), the demand for \( Y \) is constrained by income. For simplicity, we assume for now that income is always large enough so that this does not occur. Then case ii) implies that consumer \( i \)’s demand for good \( Y \) is constant and independent of her income level. After spending \( \phi_o \) on good \( Y \), she will spend all her remaining income \((\phi_i - \phi_o)\) on good \( X \) at the given price. Thus a consumer with higher income spends disproportionately more income on good \( X \) than a consumer who has lower income. This special feature of the quasi-linear utility function simplifies our analysis such that a country’s aggregate demands are independent of income distribution. We can write the Home country’s aggregate demands as follows:

\[
Y^D = \sum_{i=1}^{L} y_i = L\phi_o
\]

\[
X^D = \sum_{i=1}^{L} x_i = \sum_{i=1}^{L} \frac{\phi_i - \phi_o}{p} = \frac{I - L\phi_o}{p},
\]

where \( X^D, Y^D, y, I, \) and \( L \) are aggregate demands for good \( X \) and \( Y \), per capita consumption of good \( Y \), national income, and population, respectively. Superscript \( D \) is used to distinguish demand variables from production variables used later without a superscript. Foreign’s aggregate demands can be obtained likewise and they are independent of income distribution as well. Let us
denote the Home country’s per capita consumption of good \( X \) (without subscript \( i \)) as 
\[
x = (\phi - \phi_o)/p,
\]
where \( \phi (= I/L) \) is the Home country’s per capita income.

If we assume frictionless trade—free trade with zero transport costs—prices will be the same everywhere: \( p = p^* \), and hence \( y = y^* \). With identical preferences for consumers in Home and Foreign, we can aggregate equation (1) over all countries (Home and Foreign) and write the world market demands
\[
Y_{oW} = (L + L^*)\phi_o = L^W\phi_o,
\]
\[
X_{oW} = \frac{(I + I^*) - (L + L^*)\phi_o}{p} = \frac{I^W - L^W\phi_o}{p},
\]
where superscript \( W \) denotes world. Also note that aggregate demands for the world are determined by total income and total population of the world regardless of the distribution of income among countries or individuals.

We assume that both \( X \) and \( Y \) are produced with constant returns to scale (CRS) by competitive firms. We also assume that factors are fully employed in perfectly competitive markets and freely mobile across industries but not across countries. Profit maximizing firms will pay each factor the value of its marginal product. Then factor prices depend on factor intensities of production. With the full employment condition, factor prices in autarky will be a function of factor endowments of the country.

3. The Integrated World Equilibrium

This section provides an intuitive explanation for why we observe missing trade when we take the conventional HOV theorem to data. Suppose that countries are endowed with production factors, \( K \) and \( L \), such that Home is relatively capital-abundant and Foreign is labor-abundant. We assume that countries have the same CRS production technologies for production and that \( X \) is capital-intensive in production: \( k_c > k_y \), where \( k_c \) and \( k_y \) are capital-labor ratios of \( X \) production and \( Y \) production at common equilibrium factor prices, respectively. Together with the way
nonhomothetic preferences are defined in section 2, this factor intensity assumption captures the presumption that consumers with higher income spend disproportionately more on capital-intensive goods than lower income consumers. This presumption can be further extended to that rich countries, on average, consume capital-intensive goods more than in proportion to their world income shares, while poor countries consume labor-intensive goods more than in proportion to their income shares. In an integrated world economy (IWE) factor prices depend on the world endowments of factors. Since the endowments determine income levels of countries and a country’s population is equal to its labor force, a country with higher per capita income is the country that is endowed with more capital per capita. In our model, Home is a relatively capital-abundant country and Foreign is a relatively labor-abundant country. Hence Home is richer—with higher per capita income—than Foreign: \( k = \frac{K}{L} > k^* = \frac{K^*}{L^*} \), and therefore \( \phi > \phi^* \).

Our analysis will be confined to the inside of the factor price equalization (FPE) parallelogram, in which factor prices are equalized at any endowment point. Following the conventional notion that intraindustry trade is not based on factor endowment differences, we assume that the factor content of trade is independent of whether countries are engaged in intraindustry trade or not. We also assume that trade is balanced.

(Figure 1 about here)

Figure 1 illustrates how nonhomothetic preferences can reduce the factor content of trade in the IWE described by the Dixit-Norman-Helpman-Krugman (D-N-H-K) diagram. Let us first

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6 The way we assume for the factor intensity and nonhomothetic preferences is crucial in explaining the missing trade. This assumption will be relaxed later in the generalized model and it will be replaced with a weaker condition to derive the result.

7 See Dixit and Norman (1980) or Helpman and Krugman (1985) for explanations for the FPE parallelogram or the FPE set.

8 Davis and Weinstein (1999) contradict this notion by showing that a significant portion of intraindustry trade even among OECD countries is based on factor endowment differences in a non-integrated world. In our integrated world economy, the volume of intraindustry trade may influence the volume of factor content of trade only for the unbalanced portion of the intraindustry trade in each industry. This unbalanced intraindustry trade, however, must appear as a country’s net export or net import for the industry, which will be captured by interindustry trade. Although it is not necessary for our analysis here to distinguish the volume of intraindustry trade, this issue demands more careful attention in empirical investigations (e.g. Chung 2002).
consider the case in which factors of production are distributed across two countries such that they have the same per capita capital. Any endowment point on the diagonal line $OO^*$, point $S$ in Figure 1 for example, can serve for that purpose. Since factor endowments are the only source of income, with the above assumptions, the two countries must have the same per capita income at the given factor prices. The wage-rental ratio is denoted by (minus) the slope of the line labeled $w$ in Figure 1. Home is a smaller country with the population of $OL$, compared to Foreign whose population is measured by $O^*L^*$. How income or capital is distributed among individuals in a country will not affect our analysis as long as each consumer's income is enough to preserve the nonhomotheticity.\footnote{With too low income, it is homothetic only in an almost trivial sense, since only good $Y$ is consumed.} At the given factor prices ($w$) and production technologies ($k_x$ and $k_y$), Home will employ the vector $OM$ of capital and labor to produce $X$, the capital-intensive good, and $OC$ for $Y$ production. Let us choose units of goods so that outputs are given by the lengths of these vectors, $X=OM$ and $Y=OC$. With the endowment point $S$, Home will also consume $OM$ of good $X$ and $OC$ of good $Y$. Similarly, the production and consumption levels of $X^*$ and $Y^*$ will be $O^*M^*$ and $O^*C^*$ for Foreign. There will be no trade, or at least no net trade of factor services, because both output proportions and per capita incomes are the same for the two countries whenever endowments are on the diagonal.\footnote{Recall that HOV assumes homothetic preferences, with which countries expend on goods in proportion to their world income shares. Accordingly, consumption points under homothetic preferences will be always on the diagonal $OO^*$. On the other hand, the property of the quasi-linear utility function assures that Home, as long as the population is fixed at $OL$, will consume $OC$ of good $Y$ regardless of its income. Likewise, with its population fixed at $O^*L^*$, Foreign will consume $O^*C^*$ of $Y^*$.}

Recall that HOV assumes homothetic preferences, with which countries expend on goods in proportion to their world income shares. Accordingly, consumption points under homothetic preferences will be always on the diagonal $OO^*$. On the other hand, the property of the quasi-linear utility function assures that Home, as long as the population is fixed at $OL$, will consume $OC$ of good $Y$ regardless of its income. Likewise, with its population fixed at $O^*L^*$, Foreign will consume $O^*C^*$ of $Y^*$.

Now consider a different endowment point $E$. Moving from $S$ to $E$ is exactly the same as redistributing capital between Home and Foreign while keeping their respective populations
intact. The dotted, vertical line $LL^*$ in Figure 1 preserves the population of each country at $OL$ and $O*L^*$. As we move towards $E$ from $S$ along $LL^*$, Home becomes a relatively capital-abundant country with higher per capita income than Foreign, which turns into a relatively labor-abundant country. At the endowment point $E$, Home produces $OX$ of good $X$ and $OY$ of good $Y$. Since the population is the same as at $S$ and consumption of good $Y$ depends only on population but not on the income level, Home consumes the same amount as before of good $Y$, $OC$, and Foreign, $O*C^*$. This will be the case for any endowment point on the fixed population line, $LL^*$, as long as FPE is still achieved. At $E$, Home will export good $X$ and import exactly $YC (=Y*C^*)$ of good $Y$ from Foreign to obtain $OC$ of good $Y$ consumption. The consumption point corresponding to the endowment point $E$ will be $N$ at the given factor prices, $w$. If per capita income levels for the two countries are further apart while keeping populations at $OL$ and $O*L^*$, that is, if the endowment point is at $E'$; for example, in Figure 1, then the corresponding consumption point will be $N'$. Hence we may regard $CC^*$ as a locus of the factor content of consumption associated with nonhomothetic preferences—more specifically, QL preferences—for the endowment points on $LL^*$, in contrast to the diagonal $OO^*$, which is the corresponding locus associated with homothetic preferences. Note that $CC^*$ is parallel to the capital intensity of $X$ production, $k_x$, and that the locus associated with homothetic preferences has the slope of $k$, the capital intensity of the world endowment.

The $CC^*$ curves contingent on different population levels will all be parallel to the capital intensity of $X$ production ($k_x$) and always pass through the intersection of the diagonal ($OO^*$) and the corresponding population line ($LL^*$). As we redistribute endowments from $S$ to $E$ on $LL^*$, the consumption point moves from $S$ to $N$ along $CC^*$. Because of the particular form of preferences assumed so far, the difference between $S$ and $N$ lies only in the amount of good $X$ consumed. This is why $CC^*$ is parallel to $k_x$. If we connect the consumption point $N$ with origins $O$ and $O^*$, $ON$
will be steeper than the diagonal \((OO^*)\) while \(O^*N\) will be flatter than \(OO^*\). At the given population and factor price ratio \((w)\), therefore, the capital intensity of consumption will be higher for Home and lower for Foreign compared to the common capital intensity of consumption for both countries in the conventional HOV model. This can be interpreted as nonhomotheticity in terms of factor consumption.

This immediately leads us to a prediction of less factor content of trade (the vector \(EN\)) between the two countries than the conventional HOV prediction \((EH)\). This is in line with Markusen’s (1986) result. He showed in goods space that nonhomotheticity reduces the volume of interindustry trade between relatively capital-abundant North and labor-abundant South. Note also that the direction of the factor content of trade is preserved in our analysis— that is, capital-abundant Home still exports capital services to Foreign and imports labor services.

4. Linear Expenditure System (LES) and A More General \(CC^*\) curve

4.1. A Closed Economy

In the previous section we showed a possible reason why the conventional HOV theorem may predict too much trade, leading to the puzzle of "missing trade". Under the assumption of a quasi-linear utility function, we derived \(CC^*\) curves parallel to \(k_x\) and displayed how nonhomothetic preferences can reduce the factor content of trade. Although the analysis can be more clearly presented with the quasi-linear utility function, it is a very special version of nonhomothetic preferences. In this section, the utility function is generalized to a linear expenditure system (LES) and more general \(CC^*\) curves are generated.\(^\text{11}\) The slope range of the more general \(CC^*\) curves will be between \(k_x\) and the slope of the diagonal \((k)\), continuing the assumption that the income-elastic good is capital-intensive. A \(CC^*\) curve associated with LES differences, there will be no net factor trade.

\(^\text{11}\) The linear expenditure system is derived from a Cobb-Douglas utility function for which the origin has been displaced by the minimum consumption requirement for each good. Markusen (1986) also employed LES for nonhomothetic preferences.
(LES-CC*) will still be steeper than the diagonal (HOV-CC*)\textsuperscript{12}, but flatter than the CC* curve of the quasi-linear utility function (QL-CC*) discussed in the previous section.

We assume that preferences are identical and nonhomothetic in the form of LES. The utility function with a linear income expansion path in goods-space together with the given factor intensities in production will preserve the linearity in terms of the factor content of consumption. A consumer $i$’s utility function is given by

$$U_i = x_i^\beta (y_i - y_o)^{1-\beta}, \quad 0 < \beta < 1,$$

where $y_o$ is the minimum consumption requirement for good $Y$. Let $Y$ be the numéraire and $p$ be the price of good $X$ in terms of $Y$. From the utility maximization problem we obtain the following demand functions for consumer $i$:

$$x_i = \begin{cases} \frac{p}{\phi_i} (\phi_i - y_o) & \text{if } \phi_i \geq y_o \\ 0 & \text{if } \phi_i \leq y_o \end{cases}$$

$$y_i = \begin{cases} y_o + (1-\beta)(\phi_i - y_o) & \text{if } \phi_i \geq y_o \\ \phi_i & \text{if } \phi_i \leq y_o \end{cases}$$

Unlike the quasi-linear demand case, demand for $Y$ as well as demand for $X$ now depends on consumer $i$’s income level, $\phi_i$. Note that the parameter $\beta$ is the marginal expenditure share of good $X$. As $\beta$ approaches to unity, utility-maximizing consumer allocates more income on $X$. In the limit, equation (2) becomes exactly the same as demands we obtained in section 2 with QL preferences.\textsuperscript{13} Therefore, we can consider the results obtained with QL preferences as a special case of LES.

One can easily see from equation (2) that the income elasticity of demand for good $X$ is greater than unity while the income elasticity for good $Y$ is less than unity. The consumer’s indiffERENCE curves and an income expansion path of LES are illustrated in Figure 2.

\textsuperscript{12} Although the diagonal of the D-N-H-K diagram is labeled with $OO^*$, we will call this HOV-CC*. Since the diagonal is the locus of factor consumption associated with homothetic preferences, it is a special case of the more general CC* curve with no minimum consumption requirements.

\textsuperscript{13} Note that $\phi_o = y_o$ since good $Y$ is the numéraire.
Using the linearity of the income expansion path, we can derive aggregate demand functions as the sum of individual demand functions in equation (2). Assuming that each consumer has income of at least $y_o$, the economy’s aggregate demand for $X$ and $Y$ respectively can be obtained:

$$X^D = \frac{\beta}{p} (I - Y_o)$$
$$Y^D = Y_o + (1 - \beta)(I - Y_o), \text{ where } Y_o = L y_o. \hspace{1cm} (3)$$

The aggregate demands depend on national income ($I$), population ($L$), and the relative price ($p$), but not on income distribution among individuals.\textsuperscript{14} If the economy we consider is an integrated world with frictionless trade, equation (3) can be viewed as aggregate demands for the world with $I$ being world income and $L$ being world population. The aggregate demands of the world will be again independent of income distribution among countries, provided that each country has national income of at least $Y_o$. By dividing the aggregate demands by $L$, we can obtain per capita demand for $X$ and $Y$:

$$x = \frac{\beta}{p} (\phi - y_o)$$
$$y = y_o + (1 - \beta)(\phi - y_o). \hspace{1cm} (4)$$

Let us consider a Leontief production function for simplicity of the analysis.\textsuperscript{15} Domestic production of $X$ and $Y$ are given by

$$X = \min \left\{ \frac{K_X}{a_{kX}}, \frac{L_X}{a_{lX}} \right\}, \quad Y = \min \left\{ \frac{K_Y}{a_{kY}}, \frac{L_Y}{a_{lY}} \right\},$$

where $a_{ij}$ is a unit input requirement of factor $i=k,l$ for the production of good $j=X, Y$. $K_X, L_X, K_Y, L_Y$ are factors employed in the production of the goods. The following factor market clearing condition states that factors are fully employed:

$$K = K_X + K_Y, \quad L = L_X + L_Y, \hspace{1cm} (5)$$

\textsuperscript{14} Obviously, the assumption ($\phi_i \geq y_i \text{ for all } i$) is crucial for this simplicity.

\textsuperscript{15} The functional form chosen here helps simplify the analysis. This analysis can be also done with the more general production function.
where $K$ and $L$ are aggregate endowments of capital and labor.

Profit maximization with the Leontief production function yields

$$X = \frac{K_X}{a_{kx}} = \frac{L_X}{a_{lx}}, \quad Y = \frac{K_Y}{a_{ky}} = \frac{L_Y}{a_{ly}}. \quad (6)$$

Perfect competition in the markets of both goods and factors generates the following zero profit conditions:

$$p = ra_{kx} + wa_{lx},$$

$$1 = ra_{ky} + wa_{ly}.$$ 

Define $k_x$ and $k_y$ as capital-labor ratios of $X$ and $Y$ and assume that $X$ is relatively capital-intensive compared to $Y$, i.e., $k_x = \frac{a_{lx}}{a_{kx}} > k_y = \frac{a_{ly}}{a_{ky}}$. Then the equation can be rewritten as

$$w = \frac{1}{a_{ly}} - k_y r,$$

$$p = (k_x - k_y) + \frac{a_{lx}}{a_{ly}}. \quad (7)$$

By plugging equations (3) and (6) into the market clearing condition for good $Y$, we obtain

$$Y_o + (1 - \beta)(I - Y_o) = \frac{L_Y}{a_{ly}}. \quad (8)$$

Using equation (6), equation (5) can be rewritten as follows:

$$K_X + K_Y = K$$

$$a_{kx}X + a_{ky}Y = K \quad \text{(using } X = \frac{K_X}{a_{kx}}, \quad Y = \frac{K_Y}{a_{ky}})$$

$$\frac{a_{kx}}{a_{lx}}L_X + \frac{a_{ky}}{a_{ly}}L_Y = K \quad \text{(using } X = \frac{L_X}{a_{kx}}, \quad Y = \frac{L_Y}{a_{ky}}) \quad (9)$$

$$L_Y = \frac{k_x - k_y}{k_x - k_y} L \quad \text{(using } L = L_X + L_Y), \quad \text{where } k \equiv \frac{K}{L}.$$ 

Note that $k_x > k > k_y$ since $k$ is the capital intensity of the factor endowment. After dividing equation (8) by $L$, replacing income with factor earnings, and substituting $L_Y$ with equation (9), we can rewrite equation (8) as
\[ y_o + (1 - \beta)(w + rk - y_o) = \frac{1}{a_{y}} \frac{k_x - k}{k_x - k_y}. \]

Using equation (7), this can be rewritten for the rental rate

\[ r = \frac{(k_y - k) + (1 - a_{y,y_o})\beta(k_x - k_y)}{(1 - \beta)a_{y}(k - k_y)(k_x - k_y)}. \]  

(10)

Together with equation (7), equation (10) determines the prices of goods and factors in a closed economy for the given endowments, technology, and preferences.

4.2. Loci of the Factor Content of Consumption and Slope Analysis

Let us now turn to the slope analysis of the CC* curves contingent on different preference schemes. We can express per capita consumption of capital services and labor services as follows:

\[ c^K = xa_{ks} + ya_{ks}, \quad c^L = xa_{ls} + ya_{ls}. \]

By plugging in equation (4), we can rewrite these

\[ c^K = \frac{[\frac{\beta}{\phi}a_{ks} + (1 - \beta)a_{ks}]\phi - y_o}{\phi - y_o} + c^K_o, \]
\[ c^L = \frac{[\frac{\beta}{\phi}a_{ls} + (1 - \beta)a_{ls}]\phi - y_o}{\phi - y_o} + c^L_o, \quad \text{where} \quad c^K_o = a_{ks}y_o, \quad c^L_o = a_{ls}y_o. \]

Then the slope of LES-CC* will be

\[ \lambda^{LES} = \frac{c^K - c^K_o}{c^L - c^L_o} = \frac{\beta a_{ks} + (1 - \beta)pa_{ks}}{\beta a_{ls} + (1 - \beta)pa_{ls}}. \]  

(11)

Since HOV-CC* is a special case with zero minimum consumption, we can obtain the slope of HOV-CC* as follows:

\[ \lambda^{HOV} = \lambda^{LES} \bigg|_{y_o=0} = \frac{\beta a_{ks} + (1 - \beta)pa_{ks}}{\beta a_{ls} + (1 - \beta)pa_{ls}} \bigg|_{y_o=0}. \]  

(12)

With \( y_o=0 \), the only thing that changes in equation (12) compared to (11) is the relative price. From equation (7) and (10), the relative price of \( X \) can be obtained as
\[ p|_{s=0} = \frac{\beta a_{lx} (k_x - k)}{(1 - \beta) a_{ly} (k - k_y)}. \] (13)

Plugging equation (13) into (12) gives us

\[ \lambda_{D-H-K} = \lambda_{DES}|_{s=0} = k, \] (14)

which is the capital intensity of the endowment. This is exactly the slope of the diagonal in the D-N-H-K diagram.

We can also find the slope of QL-CC*. In the previous subsection, we discussed that QL could be a special case of LES. Recall that the parameter \( \beta \) in the LES utility is between zero and unity. As \( \beta \) approaches to unity, demands for \( X \) and \( Y \) in equation (2) approaches to demands obtained with the QL preferences. In the limit, the slope of QL-CC*, denoted by \( \lambda_{QL} \), can be

\[ \lambda_{QL} \equiv \lambda_{LES}|_{\beta=1} = \frac{a_{lx}}{a_{ly}} = k_x. \] (15)

The slope of QL-CC* is the same as the capital-labor ratio in the production of good \( X \).

Now we can show that the more general CC* curve will be anywhere between the two special CC* curves. First, let us compare the slope of LES-CC* with that of HOV-CC*. Using equations (11), (13), and (14) together with

\[ k_x = \frac{a_{lx}}{a_{lx}} > k_y = \frac{a_{ly}}{a_{ly}} \]

yields the following:

\[ \lambda_{LES} - \lambda_{HOV} = \frac{\beta a_{lx} + (1 - \beta) pa_{ly}}{\beta a_{lx} + (1 - \beta) pa_{ly}} - k \\
= \frac{\beta (k_x - k_y) a_{lx} a_{ly} y_o}{\beta a_{lx} + (1 - \beta) pa_{ly}} \\
> 0. \]

This proves that LES-CC* is steeper than HOV-CC*. Similarly, comparison of equation (11) and (15) provides us
\[
\lambda^{\text{LES}} - \lambda^{\text{QL}} = \frac{\beta a_{lx} + (1 - \beta) p a_{ly} - k_x}{\beta a_{lx} + (1 - \beta) p a_{ly}} - k_x \\
= \frac{(1 - \beta) p a_{ly} (k_x - k_x)}{\beta a_{lx} + (1 - \beta) p a_{ly}} \\
< 0.
\]

This inequality states that LES-CC* is flatter than QL-CC*.

(Figure 3 about here)

The locus of the factor content of consumption with a more general LES utility function will be a straight line and it will lie inside of a range bounded by homothetic preferences on the one end and by quasi-linear preferences on the other end. Figure 3 illustrates a locus for LES together with the bounds–bold dotted-lines–at the given per capita endowment point labeled \(v\).

Under the assumptions of the model, any CC* locus that is associated with nonhomothetic preferences will be always steeper than the locus associated with homothetic preferences. This particular property of the factor consumption loci provides an explanation for the misprediction of HOV, as already shown in section 3 with a simple diagram.

(Figure 4 about here)

Figure 4 illustrates a general CC* curve in the D-N-H-K diagram. Implications of the slope analysis for misprediction–by NH for the endowment point \(E\) and by \(N'H'\) for \(E'\)–can be easily seen from the diagram. At any endowment point in the FPE parallelogram, the LES-CC* curve can be found as a straight line passing through the intersection of the diagonal and a fixed population line, \(LL^*\). Figure 5 illustrates the LES-CC* curves corresponding to two different population sizes, \(LL^*\) and \(\bar{LL}^*\).

(Figure 5 about here)

\[16\] This is because QL is a special form of nonhomothetic preferences, which extremely restricts the consumption of a good: no more consumption of an inelastic good after a certain threshold level of income.
5. Generalization to Multi-Factors, Multi-Goods, and Multi-Countries

5.1. Income Elasticity of Factor Consumption

In this section, our discussion is generalized to arbitrary numbers of factors, goods, and countries following the spirit of Deardorff (1982). For the purpose of the generalization some alternative measures will be defined. Since income distribution under the linear expenditure system does not affect our results, as shown in the previous sections, we will consider a representative consumer, whose income is equivalent to per capita income of the country. As a first step, we will consider any number of factors and goods in a country. Hence country subscripts are omitted in the discussion of this subsection. Then later in the following subsection, our discussion will extend to any number of countries.

Suppose that a representative consumer has the LES utility function given by

\[ U(d) = \prod_{g=1}^{G} (d_g - d_{go})^{\beta_g}, \quad \sum_{g=1}^{G} \beta_g = 1, \]

where \( d_{go} \) is the minimum consumption requirement for good \( g \), which can be zero for some goods but not for all goods in order to preserve the nonhomotheticity. \( \beta_g \) is the marginal expenditure share of good \( g \). The utility maximization yields the following demand functions:

\[ d_g = d_{go} + \beta_g (\phi - \phi_o)/p_g, \quad \text{where } \phi_o = \sum_{g=1}^{G} p_g d_{go}. \quad (16) \]

\( \phi_o \) represents the amount of income needed for the representative consumer to satisfy the minimum requirements of all goods. From equation (16), income elasticity of demand is given by

\[ e_g = \frac{\beta_g \phi}{\beta_g \phi + (p_g d_{go} - \beta_g \phi_o)}. \quad (17) \]

Equation (17) shows that good \( g \) is income-elastic in its consumption if and only if the marginal expenditure share, \( \beta_g \), exceeds its share of minimum consumption, \( p_g d_{go}/\phi_o \). From this implication, a particular indicator of the income elasticity of goods consumption can be constructed as
\[ \varepsilon_g \equiv \beta_g - \frac{p_g d_{go}}{\phi_o}, \quad (18) \]

where \(\varepsilon\)'s have the convenient property of summing (over \(g\)) to zero and hence have mean zero.

Equations (17) and (18) imply that good \(g\) is income-elastic (income-inelastic) if and only if \(\varepsilon_g\) is positive (negative).

Let us now define factor intensities in the form of factor shares as in Deardorff (1982).\(^ {17}\)

Factor intensities are measured by

\[ \theta_{fg} = \frac{w_f a_{fg}}{p_g}, \quad \sum_{\ell=1}^g \theta_{f\ell} = 1, \]

where \(a_{fg}\)'s are (direct plus indirect) unit factor requirements. Using this factor intensity measure and equation (16), the consumption of factor \(f\) can be written

\[ h_f = \sum_{g=1}^G d_g a_{fg} = \sum_g d_g p_g \theta_{fg} / w_f \]
\[ = \sum_g (\beta_g \phi + p_g d_{go} - \beta_g \phi_o) \theta_{fg} / w_f \]
\[ = \frac{1}{w_f} \sum_{g} \beta_g \theta_{fg} (\phi - \phi_o + \frac{p_g d_{go}}{\beta_g}). \quad (19) \]

Since we can calculate the consumption of factor services from the unit factor requirements and goods consumption, we can also derive income elasticity of factor consumption from equation (19) as follows:

\[ \eta_f = \frac{dh_f}{d\phi} \frac{\phi}{h_f} \]
\[ = \left( \sum_{g} \beta_g \theta_{fg} / w_f \right) \frac{\phi}{h_f} \]
\[ = \frac{\sum_{g} \beta_g \theta_{fg} \phi}{\sum_{g} \beta_g \theta_{fg} \phi + \sum_{g} p_g d_{go} \theta_{fg} - \sum_{g} \beta_g \theta_{fg} \phi_o}. \quad (20) \]

\(^{17}\) Deardorff points out that the \(\theta\)'s provide a better measure of factor intensities than the \(a\)'s alone since they are unit free.
Equation (20) shows that the income elasticity of factor consumption is a function of factor intensities and income effect parameters. Note in the last row of equation (20) that the first term of the numerator is the same as the denominator. Hence, the sign of the second term of the denominator minus the third term will determine whether a factor is income-elastic or income-inelastic.

Let \( \mathbf{E} \) be a vector of length \( G \) containing income elasticity indicators for all goods and \( \mathbf{\Theta}_f \) a vector of the same length containing the factor intensities of factor \( f \) in all goods:

\[
\mathbf{E} = (e_1, \ldots, e_G) \\
\mathbf{\Theta}_f = (\theta_{f1}, \ldots, \theta_{fG}).
\]

Then the following proposition states that if these two vectors are positively correlated, the factor \( f \) is income-elastic in its consumption.

**Proposition 1.** The consumption of factor \( f \) is income-elastic (income-inelastic) if and only if

\[
\text{Cor}(\mathbf{E}, \mathbf{\Theta}_f) > 0 \quad (< 0),
\]

where \( \text{Cor}(\cdot) \) is the simple correlation between the elements of the two vectors that are its arguments.

**Proof.** Since the sign of the correlation depends on the sign of the corresponding covariance, we can prove the proposition by establishing that

\[
\eta_f > 1 \quad \text{iff} \quad \mathbf{E} \mathbf{\Theta}_f - \bar{\mathbf{E}} \bar{\mathbf{\Theta}}_f > 0,
\]

\[
(\leq) \quad (\leq)
\]

where a bar denotes mean of the vector. It follows from equation (18) that \( \bar{\mathbf{E}} \) is zero. Hence the inner product of the two vectors alone will determine whether the factor is income-elastic or not.

From (18) and (20), we can derive the following relationship between the income elasticity of factor consumption, the income elasticity of goods consumption, and factor intensities of goods production:
\[ \eta_f > 1 \quad \text{iff} \quad \sum_{g} \beta_g \theta_{fg} \phi_o > \sum_{g} p_g d_{go} \theta_{fg} \]

iff \[ \sum_{g} \left( \theta_{fg} - \frac{p_g d_{go}}{\beta_g} \right) \phi_o > 0 \]

iff \[ \sum_{g} \epsilon_g \theta_{fg} > 0. \] (22)

Since \( E \theta_f = \sum_{g=1}^{G} \epsilon_g \theta_{fg} \), this proves (21) and hence Proposition 1.

Recall that \( \epsilon_g \) is an indicator of income elasticity of good \( g \) consumption. If good \( g \) is income-elastic, \( \epsilon_g \) will be positive. For an income-inelastic good, it will be negative. Therefore, the proposition implies that a factor is income-elastic in its consumption if it is on average intensively used in goods that are income-elastic and relatively unintensively used in goods that are income-inelastic. One can easily verify the implication for an income-inelastic factor likewise.

5.2. Implications for the Factor Content of Consumption

Now our discussion turns to how the factor content of consumption of nonhomothetic preferences is compared with that of homothetic preferences. Countries are denoted by subscripts \( c = 1, \ldots, C \). Equation (19) can be written as

\[ w_{jc_f} h_{jc_f} = \sum_{g} \beta_g \theta_{fgc} (\phi_c - \phi_o) + \frac{p_g d_{go}}{\beta_g}, \]

where \( p_g \) is now the world price of good \( g \) (the world superscript \( W \) omitted). The equation is the factor content of consumption valued at the country’s factor prices. If factor prices are equalized by frictionless trade, \( w_{jc_f} = w_f \) for all countries. Then the factor intensity measure, \( \theta_{fgc} \) can be also denoted without the country subscript as \( \theta_{fg} \) for all countries since preferences and techniques of production are identical for all countries. Recall that the factor intensity measure is one of the
determinants of income elasticity of factor consumption. We need this measure to be internationally comparable for trade implications.\(^{18}\) In case of FPE, this equation can be rewritten

\[ w_{fc} = \sum g \beta_g \theta_{fg} (\phi_c - \phi_o) + \frac{p_g d_{go}}{\beta_g}. \]

With FPE and frictionless trade, we can consider the world as an IWE. Holding the world endowments fixed, prices for goods and factors will be determined by aggregate demand and aggregate production of the world. The factor prices of the IWE will be independent of cross-country income distribution because of the linearity in factor consumption.

For the comparison of the factor content of consumption for different preferences, let us construct a set of hypothetical homothetic preferences, which can lead us to exactly the same consumption of all goods as in nonhomothetic preferences at a certain endowment point. We can find this endowment point most easily by setting per capita incomes of all countries equal. Suppose that factors are proportionately distributed across countries such that per capita incomes of all countries equal to the world average level (\(\bar{\phi}\)). Then the demand per capita for good \(g\) at this income level can be written without country subscript as

\[ d_g = d_{go} + \frac{\beta_g (\bar{\phi} - \phi_o)}{p_g}. \]

Since the demand per capita is the same across all countries at this income level, we can find a marginal expenditure share for the hypothetical homothetic preferences, denoted by \(\gamma_g\), such that the demand at this particular income level equals for both preferences:

\[ d_g = \frac{\gamma_g \bar{\phi}}{p_g} = d_{go} + \frac{\beta_g (\bar{\phi} - \phi_o)}{p_g}. \]

With some manipulation, we can obtain

---

\(^{18}\) If factor prices are not necessarily equalized, an internationally comparable factor intensity measure, \(\bar{\theta}_{fg}\), can be constructed using the world-average factor price, \(\bar{w}_f\), as in Deardorff (1982). Unlike the measure
\[
\gamma_g = \beta_g - \left( \beta_g - \frac{p^d g}{\phi} \right) \frac{\phi_{g'}}{\phi} \\
= \beta_g - e_g \frac{\phi_{g'}}{\phi},
\]

(24)

where \(\sum_g \gamma_g = 1\) since \(\sum_g \beta_g = 1\) and \(\sum e_g = 0\).

Thus, we can construct the set of hypothetical homothetic preferences with this particular expenditure share, \(\gamma_g\) such that its income expansion path intersects with that of nonhomothetic preferences at the world-average income level. The factor content of consumption associated with the hypothetical homothetic preferences, denoted by \(H_{fch}\), can be defined as follows:

\[
w_j H_{fch} = \sum_g \gamma_g \theta_{fg} \phi_{g'}.
\]

(25)

Note that \(w_j\) and \(\theta_{fg}\) are without the superscript \(H\) because factor prices and goods prices are preserved with the expenditure share \(\gamma_g\). We will use this factor content of consumption contingent on hypothetical homothetic preferences to compare with that contingent on nonhomothetic preferences.

Equation (24) says that if good \(g\) is income-elastic \((e_g > 0)\), the marginal expenditure share of nonhomothetic preferences will be greater than that of hypothetical homothetic preferences:

\(\beta_g > \gamma_g\). The inequality will be reversed for income-inelastic goods. This can be visualized. Figure 6 illustrates income expansion paths with an income-elastic good, \(g'\), on the vertical axis and an income-inelastic good, \(g\), on the horizontal axis. At \(S\), where per capita income is \(\bar{\phi}\), the actual expenditure shares on \(g\) and \(g'\) under LES are the same as those under the hypothetical homothetic preferences. As suggested by the differences in slopes and origins of income expansion paths.

\[^{19}\text{In the slope analysis in section 4, different preference assumptions caused the prices of goods and factors to change. Note, however, that we construct hypothetical homothetic preferences with the expenditure shares, }\gamma_g\text{'s, obtained from the same preference assumption–LES here. We are simply hypothesizing that }\gamma_g\text{'s are marginal expenditure shares in the particular set of homothetic preferences.}\]

\[^{19}\text{In the slope analysis in section 4, different preference assumptions caused the prices of goods and factors to change. Note, however, that we construct hypothetical homothetic preferences with the expenditure shares, }\gamma_g\text{'s, obtained from the same preference assumption–LES here. We are simply hypothesizing that }\gamma_g\text{'s are marginal expenditure shares in the particular set of homothetic preferences.}\]
expansion paths, the demand for the income-elastic (income-inelastic) good, $g'$ ($g$) increases faster (slower) with nonhomothetic preferences compared to the case with homothetic preferences, where demands increase proportionately as income grows. If factor intensities of these goods differ from each other, different slopes of income expansion paths indicate that the factor content of consumption will also be different, depending on preference assumptions. We will show below how they are linked with each other and then identify what it implies for the factor content of consumption.

(Figure 6 about here)

Let us define a convenient indicator for the income elasticity of factor consumption

$$\sigma_f \equiv \Theta_f = \sum_g \epsilon_g \Theta_{fg}. \quad (26)$$

Note that this measure does not vary across countries and that its summation over all factors produces zero and hence mean zero. Equation (22) says that $\sigma_f$ is positive (negative) for income-elastic (income-inelastic) factors. By subtracting equation (25) from (23), together with equation (22) and (26), we obtain

$$w_f (h_{fc} - h_{fc}^H) = \sum_g \beta_g \Theta_{fg} \left( \phi_c - \phi_o + \frac{\mu_{icw}}{\beta_g} \right) - \sum_g \gamma_g \Theta_{fg} \phi_c$$

$$h_{fc} - h_{fc}^H = \frac{\phi_c}{\phi} (\phi_c - \phi) \sum_g \left( \beta_g - \frac{\mu_{icw}}{\beta_g} \right) \Theta_{fg} \quad (27)$$

From equation (27) we can see that the sign of the left-hand side (LHS) depends on the combined signs of the parenthesis and $\sigma_f$ of the right-hand side (RHS). In order to have a positive sign in LHS, RHS must have positives of both the parenthesis and $\sigma_f$ or negatives of both. In other words, a country’s factor content of consumption will be greater with nonhomothetic preferences than the factor consumption with homothetic preferences for an income-elastic (income-inelastic) factor if the country’s per capita income is higher (lower) than the world-average. The absolute
value of LHS for a given factor will be larger if the country’s per capita income is further apart from the world-average. This value will be also larger at the given level of income if the income elasticity of the factor is larger in absolute value, that is, if the factor is much more income-elastic or much more income-inelastic.

This suggests that there must be a general correlation between factor content of consumption—and hence trade—contingent on preferences, income levels, and income elasticity of factor consumption. In order to correlate these three variables, let us use the generalized concept of covariance, i.e., comvariance, suggested by Deardorff (1982) since covariance is defined only for two variables. Let \( \mathbf{x}, \mathbf{y}, \) and \( \mathbf{z} \) be vectors of length \( N \). Then the comvariance among the three vectors is the following:

\[
\text{Com}(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \sum_{i=1}^{N} (\bar{x}_i - \bar{x})(\bar{y}_i - \bar{y})(\bar{z}_i - \bar{z}),
\]

where a bar over a variable denotes its mean.

Let us denote the gap between nonhomothetic factor consumption and homothetic factor consumption by

\[
\omega_{fc} = h_{fc} - h_{fc}^H = \frac{\phi_c (\bar{\phi}_c - \bar{\phi})}{\bar{w}_f \bar{\phi}} \sigma_f.
\]

Let \( \Phi, \Sigma, \) and \( \Omega \) be vectors of length \( M=CF \) containing per capita incomes of all countries, income elasticity measures of all factors, and the factor consumption gaps (\( \omega_{fc} \)'s) for each country and factor, respectively. Hence, the elements of these vectors are \( \phi_c \) for all \( f \), \( \sigma_f \) for all \( c \), and \( \omega_{fc} \).

Then the following proposition shows the correlation between the three variables.

**Proposition 2.** If preferences are identical and nonhomothetic, and techniques of production are identical across countries, then, as long as per capita incomes are not identical for all countries,

\[
\text{Com}(\Phi, \Sigma, \Omega) > 0.
\]
**Proof.** Applying equation (28) to (30), we can derive the following:

\[
\text{Com}(\Phi, \Sigma, \Omega) = \sum_{m=1}^{M} (\phi_m - \bar{\phi})(\sigma_m - \bar{\sigma})(\omega_m - \bar{\omega}) \\
= \sum_{f=1}^{F} \sum_{c=1}^{C} \phi_{fc} \sigma_{fc} \omega_{fc} + 2M\bar{\phi} \bar{\sigma} \bar{\omega} - \bar{\phi}(\Sigma \cdot \Omega) - \bar{\sigma}(\Phi \cdot \Omega) - \bar{\omega}(\Phi \cdot \Sigma).
\] (31)

Since \( \sum_{f=1}^{F} \theta_{fc} = 1 \), it follows from equation (26), (27), and (29) that \( \bar{\sigma} = 0 \), and \( \bar{\omega} = 0 \). Together with these mean values, let us use notation that \( \phi_{fc} = \phi_c \) for all \( f \), \( \sigma_{fc} = \sigma_f \) for all \( c \). Then equation (31) can be written as follows:

\[
\text{Com}(\Phi, \Sigma, \Omega) = \sum_{f=1}^{F} \sum_{c=1}^{C} \phi_{fc} \sigma_{fc} \omega_{fc} + 2M\bar{\phi} \bar{\sigma} \bar{\omega} - \bar{\phi}(\Sigma \cdot \Omega) - \bar{\sigma}(\Phi \cdot \Omega) - \bar{\omega}(\Phi \cdot \Sigma).
\]

The case of identical per capita income for all countries is excluded in the proposition because the income effect cannot be captured in that case. Average per capita consumption in that case will be the same across countries regardless of whether preferences are nonhomothetic or homothetic. Excluding this case, the proof shows that the proposition holds with strict inequality.

For better interpretation of the result, let us rewrite equation (30) as

\[
\text{Com}(\Phi, \Sigma, \Omega) = \sum_{m=1}^{M} (\phi_m - \bar{\phi})(\sigma_m - \bar{\sigma})(\omega_m - \bar{\omega}) > 0,
\]

where \( m \) represents a particular factor-country combination for which there are values of per capita income, income elasticity of factor consumption, and factor content of trade. Deardorff (1982) provides an analysis of how to interpret this convariance. Following his mathematical interpretation, the above result suggests that for the summation to be positive, the \( M \) terms must

---

20 For the etymological reference of convariance, see Deardorff (1982) p. 690, footnote 10.
on average be positive. For the product for a particular \( m \) (that is each term) to be positive, either all three factors must be positive or exactly two must be negative.

Now let us turn to the economic interpretation of this result. It says that consumers in relatively rich countries (\( \phi_m > \bar{\phi} \)) must on average consume disproportionately more (\( \omega_m > \bar{\omega} = 0 \)) of *income-elastic* factors (\( \sigma_m > \bar{\sigma} = 0 \))—that is, factors that are on average used relatively intensively in income-elastic goods (\( \sum_{g} \varepsilon_{fg} \theta_{fg} > 0 \))—and must consume disproportionately less (\( \omega_m < \bar{\omega} = 0 \)) of *income-inelastic* factors (\( \sigma_m < \bar{\sigma} = 0 \)). Likewise, in relatively poor countries (\( \phi_m < \bar{\phi} \)), consumers must on average consume *income-elastic* factors disproportionately less and *income-inelastic* factors disproportionately more.

### 6. Implications for “Missing Trade”

The difference between factor content of consumption with nonhomothetic preferences and that with homothetic preferences is particularly interesting because of its implications for the missing trade. A simple modification to Proposition 2 enables us to extract direct implications of different preferences for the factor content of trade. These implications help us understand why we observe missing trade between countries with different income levels.

Let us define an indicator for the total volume of the factor content of trade for country \( c \) as the following:

\[
T^H_c = \sqrt{\sum_f (t^H_{fc})^2}, \quad T^N_c = \sqrt{\sum_f (t^N_{fc})^2},
\]

where \( t^H_{fc} = v_{fc} - q_c h^H_{fc} \), \( t^N_{fc} = v_{fc} - q_c h^N_{fc} \).

Superscripts \( H \) and \( N \) denote homothetic preferences and nonhomothetic preferences, respectively. \( t_{fc} \) and \( h_{fc} \) are the factor content of trade and per capita consumption of factor \( f \) in country \( c \), respectively. \( v_{fc} \) represents the endowment of factor \( f \) in country \( c \), and \( q_c \) denotes
Proposition 3. Suppose that factors can be divided into two groups: \( F = K \cup L \), where \( F \) is the set of all factors. \( K \) is a subset of factors that are abundant in relatively rich countries and scarce in relatively poor countries. \( L \) is the other subset of factors that are abundant in relatively poor countries and scarce in relatively rich countries. If the assumptions of Proposition 2 are satisfied and if \( \text{Cor}(\Theta_f, E) > 0 \) for all \( f \in K \) and \( \text{Cor}(\Theta_f, E) < 0 \) for all \( f \in L \), then

\[
T_c^N < T_c^H \quad \text{for all } c.
\]

Proof. We can prove the proposition by establishing that \( T_c^N - T_c^H < 0 \). Since both \( T_c^N \) and \( T_c^H \) are positive, the sign of \( (T_c^N - T_c^H) \) is the same as the sign of \( \{ (T_c^N)^2 - (T_c^H)^2 \} \).

\[
(T_c^N)^2 = \sum_f (t_{fc}^N)^2 = \sum_f (v_{fc} - q_c h_{fc}^N)^2 = \sum_f \{(v_{fc})^2 - 2 q_c v_{fc} h_{fc}^N + (q_c h_{fc}^N)^2 \}
\]

\[
(T_c^H)^2 = \sum_f (t_{fc}^H)^2 = \sum_f (v_{fc} - q_c h_{fc}^H)^2 = \sum_f \{(v_{fc})^2 - 2 q_c v_{fc} h_{fc}^H + (q_c h_{fc}^H)^2 \}
\]

Using above equations together with equation (27), we can obtain

\[
(T_c^N)^2 - (T_c^H)^2 = -2 q_c \sum_f v_{fc} (h_{fc}^N - h_{fc}^H) + q_c^2 \sum_f \{(h_{fc}^N)^2 - (h_{fc}^H)^2 \}
\]

\[
= -2 q_c \sum_f v_{fc} (h_{fc}^N - h_{fc}^H) + q_c^2 \sum_f (h_{fc}^N + h_{fc}^H) (h_{fc}^N - h_{fc}^H)
\]

\[
= -q_c \sum_f (h_{fc}^N - h_{fc}^H) (2 v_{fc} - q_c h_{fc}^N - q_c h_{fc}^H)
\]

\[
= -q_c \sum_f (h_{fc}^N - h_{fc}^H) (2 \alpha_{fc})
\]

\[
= -2 q_c \frac{\phi_o}{\phi} (\phi_c - \bar{\phi}) \sum_f \frac{\sigma_f \alpha_{fc}}{w_f}.
\]
where \( \alpha_{fc} \equiv \frac{(v_{fc} - q_h^f)(v_{fc} - q_h^f)}{2} \) is an indicator for factor abundance.\(^{21}\) The sign of the above equation can be verified by examining the assumptions of the proposition. If \( f \in L \), then factor \( f \) must be income-inelastic \( (Cor(\Theta_f, \Theta) < 0, \text{i.e., } \sigma_f \equiv E\Theta_f = \sum \varepsilon_g \theta_{fc} < 0 ) \). By definition, it must be abundant \( (\alpha_{fc} > 0) \) in a relatively poor country \( (\phi_c < \phi) \), and scarce \( (\alpha_{fc} < 0) \) in a relatively rich country \( (\phi_c > \phi) \). Likewise, if \( f \in K \) (i.e., \( \sigma_f > 0 \)), it must be abundant \( (\alpha_{fc} > 0) \) in a relatively rich country \( (\phi_c > \phi) \), and scarce \( (\alpha_{fc} < 0) \) in a relatively poor country \( (\phi_c < \phi) \). Thus, whenever \( \phi_c < \phi \), it is true that \( \sigma_f \alpha_{fc} < 0 \) for all \( f \) in country \( c \). Likewise, whenever \( \phi_c > \phi \), \( \sigma_f \alpha_{fc} > 0 \) for all \( f \) in this relatively rich country. Therefore, we can find \( (\phi_c - \phi)\sum \sigma_f \alpha_{fc} > 0 \). Together with the minus sign in front, \( (T_c^N)^2 - (T_c^H)^2 < 0 \), and hence \( T_c^N < T_c^H \).

Recall that \( \overline{\phi} \) is the world-average per capita income, where the income expansion path for LES with the marginal expenditure share, \( \beta \), intersects with the income expansion path for HOV with \( \gamma \). Figure 6 illustrates this in the case of two goods. If a country's per capita income is above this world-average income level, nonhomothetic preferences will induce the country to spend more on income-elastic goods and less on income-inelastic goods than do the corresponding homothetic preferences.

Let \( \Phi, \Sigma, \text{ and } A \) be vectors of length \( M = CF \) containing per capita incomes of all countries, indicators for income elasticity of all factors, and indicators for factor abundance of all factors for all countries, respectively. Then the difference in factor content of trade predicted by homothetic

\(^{21}\) With this indicator, factor abundance is defined as follows; a factor is abundant in a country if the endowment of the factor exceeds the average of two factor contents of consumption predicted by nonhomothetic preferences and by homothetic preferences.
preferences and nonhomothetic preferences for the world as a whole can be summarized in following corollary.

**Corollary 1.** If $\text{Com}(\Phi, \Sigma, \mathbf{A}) > 0$, then

$$T^N < T^H,$$

where $T^N = \sqrt{\sum_c \sum_f (t^N_{fc})^2}$ and $T^H = \sqrt{\sum_c \sum_f (t^H_{fc})^2}$.

**Proof.** We can prove the corollary by establishing that $T^N - T^H < 0$. Since both $T^N$ and $T^H$ are positive, again it is equivalent to showing that $(T^N)^2 - (T^H)^2 < 0$. From the proof of Proposition 3, we can obtain

$$(T^N)^2 - (T^H)^2 = -2 \frac{\phi_f}{\phi} \sum_c (\phi_c - \bar{\phi}) \sum_f \frac{\sigma_f \alpha_{fc}}{w_f}.$$

Since $q_c$ and $w_f$ are strictly positive, the sign of $(T^N)^2 - (T^H)^2$ depends on the sign of $- \sum_c \sum_f (\phi_c - \bar{\phi}) \sigma_f \alpha_{fc}$. By definition, the comvariance can be expressed as

$$\text{Com}(\Phi, \Sigma, \mathbf{A}) = -\sum_{m=1}^M (\phi_m - \bar{\phi}) (\sigma_m - \bar{\sigma}) (\alpha_m - \bar{\alpha}) > 0,$$

where $m$ represents a particular factor-country combination, $f c$. Then $\phi_m = \phi_c$ for all $f$, $\sigma_m = \sigma_f$ for all $c$, and $\alpha_m = \alpha_{fc}$. Together with zero mean values ($\bar{\sigma} = \bar{\alpha} = 0$), the comvariance can be rewritten as $\text{Com}(\Phi, \Sigma, \mathbf{A}) = \sum_c \sum_f (\phi_c - \bar{\phi}) \sigma_f \alpha_{fc} > 0$. Thus, $(T^N)^2 - (T^H)^2 < 0$ and this proves Corollary 1.

Corollary 1 says that HOV overpredicts factor content of trade for the world as a whole compared to the model with nonhomothetic preferences, under the condition

$$\text{Com}(\Phi, \Sigma, \mathbf{A}) = \sum_c \sum_f (\phi_c - \bar{\phi}) (\sigma_f - \bar{\sigma}) (\alpha_{fc} - \bar{\alpha}) > 0.$$
For the result of Corollary 1, therefore, the condition we need is that income-elastic factors \((\sigma_f > \bar{\sigma} = 0)\) are on average abundant \((\alpha_{fc} > \bar{\alpha} = 0)\) in rich countries \((\phi_c > \bar{\phi})\) and on average scarce \((\alpha_{fc} < \bar{\alpha} = 0)\) in poor countries \((\phi_c < \bar{\phi})\). Likewise, income-inelastic factors \((\sigma_f < \bar{\sigma} = 0)\) must on average be scarce \((\alpha_{fc} < \bar{\alpha} = 0)\) in rich countries \((\phi_c > \bar{\phi})\) and on average abundant \((\alpha_{fc} > \bar{\alpha} = 0)\) in poor countries \((\phi_c < \bar{\phi})\).

The following remark summarizes the results of this section and restates their implications for the missing trade.

**Remark 1.** Proposition 3 and Corollary 1 show that the conventional HOV theorem predicts factor content of trade excessively for individual countries and for the world as a whole compared to nonhomothetic preferences. This result holds to the extent that income-elastic factors are on average abundant in rich countries and income-inelastic factors are on average abundant in poor countries. Thus, missing trade can in part be explained by incorporating nonhomothetic preferences into the Heckscher-Ohlin-Vaněk model.

7. **Conclusion**

This paper provides another explanation for why we observe missing trade from the demand side by incorporating nonhomothetic preferences into the conventional HOV model. In a simple model with two factors, two goods, and two countries, we showed that the HOV theorem over-predicts the factor content of trade compared to the model with nonhomothetic preferences, if the consumption of capital services is income-elastic and that of labor services is income-inelastic. This overprediction of HOV was clearly illustrated in a diagram with the loci of the factor content of consumption–CC* curves–contingent on different preferences. From the slope analysis of the
CC* curves, we also showed that, with nonhomothetic preferences, the capital intensity of factor consumption increases as income rises, while it remains constant if preferences are homothetic.

By extending the model to arbitrary number of goods, factors, and countries, we provide a more generalized setting that can be more appropriate for empirical work than the simple model. First, we defined income elasticity of consumption of factor services by combining factor intensity of goods production and income elasticity of goods consumption. According to this definition, the consumption of a factor will be income-elastic if the factor is, on average, used intensively in production of income-elastic goods and relatively unintensively in goods that are income-inelastic. Second, we investigate the relationship between per capita income, the income elasticity of factor consumption, and the factor content of consumption. Our model consistently predicts that relatively rich countries, on average, consume services of income-elastic factors more than proportionately and services of income-inelastic factors disproportionately less. Third, we devised indicators that measure the volume of factor content of trade for each country and for the world as a whole. Direct comparison of these indicators for each model shows that HOV predicts more factor content of trade for any country and for the world as a whole than does the model with nonhomothetic preferences. This result holds to the extent that income-elastic factors are, on average, abundant in rich countries and income-inelastic factors are, on average, abundant in poor countries.

Despite its long history of explaining why countries trade and its elegant theoretical prediction for the pattern of trade, more recently, the HOV model has been more frequently cited for its empirical ineffectiveness. On the top of the list stands the missing trade. Trefler (1995) first reported the case of the missing trade together with other anomalies. Gabaix (1997) also reconfirmed the phenomenon of missing trade. Recall that the missing trade refers to the special pattern of data such that the measured factor content of trade is too small vis-à-vis the theoretical prediction. We suggest that one possible cause of the missing trade is misprediction stemming
from the strong assumption of homothetic preferences. This paper has shown that the missing trade can in part be attributed to the excessive prediction of the HOV model.
References


Figure 1. Locus of the Factor Content of Consumption and Trade Prediction: Quasi-Linear Utility
Figure 2. Income Expansion Path of Linear Expenditure System (Source: Markusen (1986))
Figure 3. Loci of the Factor Content of Consumption: $CC^*$ curves
Figure 4. Locus of Factor Content of Consumption under Linear Expenditure System
Figure 5. CC* Curves contingent on the distribution of population
Figure 6. Income expansion paths for nonhomothetic preferences and hypothetical homothetic preferences